Titles and Abstracts

International conference and workshop on surfaces of infinite type

29 July–2 August 2013

Minicourses

Wind tree models: Samuel Lelièvre and Vincent Delecroix

Wind-tree models are 2D Boltzmann gas models where a point-particle moves at unit speed with perfect bounces off rectangular obstacles in the plane. Paul and Tatiana Ehrenfest introduced them in 1912 with randomly placed obstacles; in the 1980s J. Hardy and J. Weber introduced lattice-periodic wind-tree models: obstacles at integer points, sides horizontal and vertical.

By relating these models to infinite-area translation surfaces and using the dynamics of genus two translation surfaces, explored and explained in great detail in recent years, one can gain insight into the following dynamical questions:

- Does a generic trajectory come back arbitrarily near its starting point? (recurrence versus transience)
- How fast does a particle explore the billiard? (diffusion)
- Do particles explore every portion of the space? (minimality and ergodicity)

Infinite dimensional Teichmüller spaces: Alastair Fletcher

Teichmüller theory is an important area of modern mathematics with links to many other subjects. Much of the theory focusses on Riemann surfaces of finite analytic type, but in this mini-course we will survey results in Teichmüller theory which hold for all Riemann surfaces, and in particular in the case of infinite analytic type. We will discuss the topics including:

- Quasiconformal mappings and their basic properties;
- Teichmüller space, its complex structure, tangent space;
- Biholomorphic maps between Teichmüller spaces;
- The geometry of Teichmüller space, extremal and uniquely extremal maps, geodesics;
- The bi-Lipschitz structure of Teichmüller space.

Lectures

David Aulicino: *Some results on affine manifolds in genus 3 with a zero Lyapunov exponent*

I will introduce the Lyapunov exponents of the Teichmüller geodesic flow on the moduli space of Abelian differentials and give some background on what is known about them. In genus 3, there are at most two Lyapunov exponents of the Kontsevich–Zorich cocycle that are non-trivial in the sense that they cannot be explicitly computed for all affine invariant manifolds. It was proven that there is exactly one surface whose $\mathrm{SL}(2, \mathbb{R})$ orbit has the property that both of these exponents are zero. We consider the problem of exactly one of these exponents being zero and present progress on what is known.

Ara Basmajian: *Laminations on general hyperbolic surfaces*

Let S be a hyperbolic surface. A (geodesic) lamination on S is a disjoint union of simple geodesics which form a closed subset of S; the individual geodesics of the lamination are called the leaves of the lamination. We say that a lamination L has the (dense closed leaves) DCL property if every open leaf of L is the limit of closed leaves. If S has finite topological type then such a lamination has a simple structure, namely it is the finite disjoint union of simple closed geodesics. In this talk we present preliminary results on a study of DCL-laminations on infinite type surfaces where they have a complex, rich structure.

Simion Filip: *Some rigidity properties of the Kontsevich–Zorich cocycle*

The behavior of the Kontsevich–Zorich (KZ) cocycle controls many properties of the dynamics on individual flat surfaces. This link has been used to prove many strong results, but they usually apply to almost every flat surface and do not give information about a particular example. I will discuss some results about the continuity of invariant splittings of the KZ cocycle. Combined with measure classification results of Eskin–Mirzakhani this implies, for example, that the Oseledets theorem holds for all flat surfaces in a.e. direction (before, it was only a.e. flat surface). Time permitting, I will also discuss some results reminiscent of the Deligne semisimplicity theorem (but in the context of the KZ cocycle).

Jesús Hernández: *Curve complexes of infinite type surfaces*

In this talk we will first introduce and give a brief review of the works in curve complexes and mapping class groups of finite type surfaces; then we will introduce the techniques we used for the analogous results in infinite type surfaces along with conjectures, questions and problems derived from these.

Pat Hooper: *Deforming infinite flat surfaces*

The dynamics of the straight-line flow on closed translation surfaces is intimately connected to the geometry of Teichmüller space equipped with its Teichmüller metric. It is reasonable to expect that the same should hold for the dynamics of the straight line flow on translation surfaces of infinite topological type. Therefore, it is reasonable to be interested in the geometry and topology of the space of translation surfaces of infinite topological type. I'll discuss some examples of limits and continuously varying families of translation surfaces of infinite type. I'll discuss some relationships between these families and other topics such as billiard problems and ergodic theoretic questions for the straight-line flow. Such discussions of limits and continuity involve an informal feeling for the topology on the space of all translation surfaces (closed or not). I'll describe some work in progress attempting to formalize a useful topology on this space.

Chris Johnson: *Hyperelliptic Translation Surfaces and Panov Planes*

Motivated by a construction of Dmitri Panov, we show how to associate a collection of half-translation planes to any hyperelliptic translation surface, and study the geodesics in eigendirections of affine pseudo-Anosov's on the surface. This construction can be used to study dynamics in the periodic Ehrenfest wind-tree model.

Erina Kinjo: *On the length spectrums of Riemann surfaces of infinite type and Teichmüller metric*

We consider the length spectrum metric in infinite dimensional Teichmüller space $T(R)$. It is known that the length spectrum metric defines the same topology as that of the Teichmüller metric on Teichmüller space of any Riemann surface of finite type. In 2003, H. Shiga proved that the two metrics define the same topology on $T(R)$ if R is a Riemann surface which can be decomposed into pairs of pants such that the lengths of all their boundary components except punctures are uniformly bounded by some positive constants from above and below.

In this talk, we extend Shiga's result to Teichmüller spaces of Riemann surfaces satisfying a certain geometric condition. Also we give a sufficient condition for the two metrics to define the different topologies.

Kathryn Lindsey: *Infinite type flat surfaces via Bratteli diagrams and "cutting and stacking"*

I will present a way of viewing a class of infinite type flat surfaces as suspensions of "cutting and stacking" transformations of the unit interval. These surfaces may be parameterized by compatibly weighted Bratteli diagrams (a type of infinite directed graph with associated combinatorial data); any such Bratteli diagram determines a surface. Some dynamical properties of the surface may be "read off" the Bratteli diagram. An application of a theorem by Treviño yields a condition on the Bratteli diagram that guarantees unique ergodicity of the vertical flow on the associated flat surface. This approach links the studies of infinite type translation surfaces and rank r transformations of the unit interval, while also providing a large class of examples of infinite type flat surfaces. This is joint work with R. Treviño.

Robert Niemeyer: *Sequences of translation surfaces determined from sequences of prefractal (rational) billiard tables*

In this talk, we will first discuss recent results concerning periodic orbits and nontrivial paths of fractal billiard tables. It is conjectured that such fractal billiard tables have associated 'fractal translation surfaces'. We will discuss three examples of fractal billiard tables, each of which has a corresponding sequence of translation surfaces that presumably converges to what should be the associated 'fractal translation surface'. The examples we consider are the Koch snowflake fractal, a self-similar Sierpinski carpet and what we are calling the "T-fractal", really for lack of a better name. We will discuss possible approaches to 1) constructing such a surface and 2) determining a well-defined flow on such a surface. The ultimate goal of such a research program is to show that 1) there is a billiard flow on a fractal billiard table F, 2) such a billiard flow is equivalent to the geodesic flow on the associated 'fractal translation surface' $S(F)$ and 3) that classical results in the subject of rational billiards have analogues for billiard tables with fractal boundary.

Daniel Pellicer: *Maps to get to the Loch Ness Monster*

A map on a surface S is a collection of points of S, called vertices; and line segments on S between pairs of vertices, called edges; with the property that when we remove from S all vertices and edges, the connected components of the remaining topological space, called faces, are homeomorphic to discs.

Maps on surfaces have been deeply studied since the beginning of the 20th century. Most of their study is on finite maps, or equivalently, of maps on compact surfaces. Recently, attention has been given to infinite maps like the tessellations of the Euclidean plane.

In this talk we show how some particular tessellations of the plane, called Archimedean tilings, naturally induce maps in the non-compact surface called the Loch Ness Monster. Geometry, algebra, topology and combinatorics are combined to show that the so-called minimal regular covers of many tessellations of the Euclidean plane are maps on this non-compact surface.

David Ralston: *(Topologically) generic ergodicity of periodic group extensions over translation surfaces*

Periodic group extensions may be formed over a translation surface M by choosing geodesic segments on M which act to translate the geodesic flow by a fixed amount within a locally compact Abelian group G, creating a geodesic flow on the product of M and G. In past work joint with S. Troubetzkoy, we showed that certain surfaces have the property that almost every such construction has the property that the geodesic flow is ergodic in almost every direction. Continuing this work, still joint with S. Troubetzkoy, we will show that within each stratum of compact translation surfaces, there is a residual (dense G_δ) set of such surfaces.

Anja Randecker: *Singularities on infinite translation surfaces*

Finite translation surfaces are surfaces with a translation atlas defined on the whole surface but finitely many points such that the metric completion w.r.t. this atlas is a compact surface. The finitely many points are called singularities and each of them has a neighbourhood that is isometric to a k-fold translation covering of a neighbourhood of $0 \in \mathbb{C}$ (for $k \in \mathbb{N}$).

When we pass to infinite translation surfaces, k can be ∞, but even more can happen: one can have *wild* singularities. Such a singularity does not have a neighbourhood that can be understood as a covering of something elementary. In "Wild singularities of flat surfaces" Joshua Bowman and Ferrán Valdez proposed a way to describe these singularities.

In my talk I will speak about this description and show how it can be used to better understand infinite translation surfaces.

Rodrigo Treviño: *Controlling deforming geometries*

Masur's criterion for unique ergodicity for translation flows on flat surfaces says that if the Teichmüller orbit of a flat surface is recurrent to a compact set of the moduli space, then the translation flow defined by that surface is uniquely ergodic. The recurrence to a compact set in this criterion amounts to the geometry of the surface not degenerating as one applies the Teichmüller deformation.

I will present a theorem which applies to any flat surface of finite area which says that if the geometry of a flat surface undergoing Teichmüller deformation can be more or less controlled, then the translation flow is uniquely ergodic. In the case of the surface being compact, this implies Masur's criterion. The proof is inspired by Forni's proof for the spectral gap of the Kontsevich–Zorich cocycle. I will discuss the proof (which is in my opinion the more magical part of this) and discuss some possible future directions of this approach.