Transcontinental Seminar with actual topic: "CAT(0)-Geometry and Classifying spaces".

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In the fifth edition of the Transcontinental Seminar we will focus on $CAT(0)$ geometry and its relevance to construct classifying spaces for families.

The main objective of the seminar is to understand the recent work of Farley [Far09], Lück [Lüc09], and Pineda-Leary [JPL06] giving concrete descriptions of the classifying space for the families of finite, respectively virtually cyclic subgroups inside a discrete group.

The main result we will understand is theorem 5.1 in [Far09], stating that under some assumptions (existence of well-behaved axes), a certain join construction of classifying spaces associated to coverings of the axes, respectively, of the CAT(0)-space provide a model for the classifying space of virtually cyclic groups inside a CAT(0)-group.

These spaces are relevant to computations and proofs of results concerning the Baum-Connes and the Farrell-Jones Conjectures [LR05].

The program features the participation of Luis Jorge Sánchez as local organiser in Morelia, who will be responsible of coordinating the activities of the mexican team, particularly organizing the informal discussion session in Morelia.

The Seminar is divided in three tematic blocks:

- Classifying Spaces and the glueing method.
- Basic $CAT(0)$-Geometry.
- Proof of the main theorem.
- Classifying Spaces and the glueing method. The main goal of this section is to recall the basic definitions of classifying spaces for groups, classifying spaces for families, the nerve of a covering, as discussing some results which allow to obtain the classifying space for virtually cyclic subgroups from the classifying space for proper actions. This part will consist of lectures:

    1. Classifying spaces for groups and for coverings. In this talk, the speaker shall introduce the notion of the classifying space of a group via the bar construction, as well as the classifying space of a covering, [Seg68]. Prerequisites are of course some comments on simplicial sets and their realization. For the informal discussion, the paper of segal [Seg68] is highly recommended to be taken a look of, specially the

construction of the spectral sequence. Anothner interest topic may be the relation to the Join constuction.

2. Classifying spaces for families. This talk aims to give the definition of the classifying space for a familiy of subgroups, and to specialize in the classifying spaces for proper actions $\underline{E}G$ and the classifying space for the family of virtually cyclic subgroups $E_{\mathcal{VC}}(G)$. Literature: [Lüc05], section 1 . The talk must include a discussion of the two models of $\underline{E}Sl_2(\mathbb{Z})$ (viewed as a tree, an as the isometries of the hyperbolic plane), as well as the construction of $E_{\mathcal{VC}}(G)$ due to Quinn-Farrell (these examples are discussed in W.L lecture slides from WS 2009, pages 21-27. For the informal session, the relation between several universal models for $\underline{E}G$ may be studied (numerable version, operator theoretical model).
3. The glueing method. This talk should explain how to obtain the classifying space for families from other simple ones This should include section 2 in [Far09]. Other important examples are Groups with conditions N, NM for $\underline{E}G$ of Lück, and if possible some example like [JPL06] or [LW07] for $E_G(\mathcal{VC})$, without proof. The informal session should aim to treat some details of this proof.

- Basic $CAT(0)$-Geometry. The aim of this section is to introduce standard tools from $CAT(0)$-geometry which are needed for the proof of the main theorem. This includes particularly the flat stripe theorem and the proper decomposition theorem , as well as the uniqueness of geodesics and its consequences. The main result in this section is that CAT(0)-spaces provide an example of a classifying space for proper actions.

  1. Generalities on CAT(0-spaces, Uniqueness of geodesics, Convexity of balls , unique centres for balls. References:[BH99]. Examples of $CAT(0)$-spaces.
  2. Convexity and its consequences. This talk should ennunciate the main theorems in Chapter II.2 in [BH99], the flat stipe theorem and the product decomposition theorem [BH99] theorem 2.13 and 2.14. On the way to it, the existence of centres and projections for convex subspaces shall be discussed. The talk should finalize with the proof that a $CAT(0)$ space acted on properly and cocomplactly by a group G is a model for $\underline{E}G$, corollary 2.8, page 179 in [BH99].

- Proof of the main theorem. In this section, we shall go into the details of the proof of Theorem 5.3 in [Far09].

  1. Spaces of lines. In this talk the lecturer will use the flat strip theorem to prove that in the "Well behaved case", the spaces $\mathcal{A}(X,\Gamma)$ are a union of proper $CAT(0)$-spaces. This should include the examples in section 7 of [Far09] without proof (spaces with isolated flats and real analytic manifolds of nonpositive sectional curvature. References: section 3 in [Far09] , as well as counterexample 3.11
  2. Covers for $CAT(0)$- groups. the purpose of this talk is to introduce the notions of "good covers"', section 4 in [Far09]. The crucial point is the proof of proposition 4.6 in [Far09].

3. Proof of the main theorem. In this talk, the proof of theorem 5.3 will be completed. References: section 5 in [Far09].

References

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[LW07] W. Lück and M. Weiermann. On the classifying space of the family of virtually cyclic subgroups. Preprintreihe SFB 478 — Geometrische Strukturen in der Mathematik, Heft 453, Münster, arXiv:math.AT/0702646v2, to appear in the Proceedings in honour of Farrell and Jones in Pure and Applied Mathematic Quarterly, 2007.

[Seg68] Graeme Segal. Classifying spaces and spectral sequences. *Inst. Hautes Études Sci. Publ. Math.*, 34:105–112, 1968.