

Prospectos en Topología

SEMESTER 2024-1

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During the 2024-1 term, the Seminar “Prospectos en Topología” will have the following two thematic blocks:

1. Bounded Cohomology of Topological Groups.
2. Hyperbolic Structures on Groups.

The seminar will also feature a number of individual talks. The schedule for the seminar is as follows:

- Bounded Cohomology of Topological Groups

Objective: To review the basics of bounded cohomology of topological groups. In particular, we will focus on group-theoretical properties reflected in bounded cohomology such as amenability or Kazhdan’s Property (T).

1. Introduction to Bounded Cohomology of Topological Groups
 - **Speaker:** Noé Bárcenas.
 - **Dates:** August 7th and August 14th.
 - **References:** [Löh10] and [Mon06].
2. Amenability in Terms of Bounded Cohomology
 - **Speaker:** Carlos Pérez Estrada.
 - **Date:** August 21st.
 - **References:** [Mon01], [dlH79] and [GdlH15].
3. Bounded Cohomology of Big Mapping Class Groups
 - **Speaker:** Israel Morales.
 - **Date:** September 18th.

- Individual Talk:

- **Speaker:** Anja Randecker.
- **Date:** August 28th.

- Individual Talk: “Strong Topological Rigidity of Non-Compact Orientable Surfaces”

- **Speaker:** Sumanta Das.
- **Date:** September 11th.
- **Reference:** [Das23].

- Individual Talk: “Magnitude Homology and Topological Data Analysis”

- **Speaker:** Haydeé Contreras Peruyero.
- **Date:** October 2nd.

- Individual Talk: “Big Braid Groups”

- **Speaker:** Daniel Juan-Pineda.
- **Date:** October 9th.

- Hyperbolic Structures on Groups

Objective: To study the general theory of hyperbolic structures on groups introduced by Carolyn Abbott, Sahana H. Balasubramanya and Denis Osin in [ABO19]. Specifically, we will introduce the notion of geometric structures for (non-necessarily finitely generated) groups and we will study the poset structure of the families of hyperbolic structures and acylindrically hyperbolic structures.

1. Poset Structure of the Family of Linear Hyperbolic Structures

- **Speaker:** Carlos Pérez Estrada.
- **Date:** October 16th.
- **Abstract:** We will start setting up the preliminaries about hyperbolic structures on groups. Then we will prove Theorem 2.3 of [ABO19] concerning the poset structure of the subfamilies of linear hyperbolic structures and orientable linear hyperbolic structures on arbitrary groups.
- **References:** [ABO19].

2. Acylindrically Hyperbolic Groups

- **Speaker:** Jesús Hernández Hernández.
- **Date:** October 30th.
- **Abstract:** Acylindrically hyperbolic groups are introduced following [Osi16] and a characterization of them presented in [Osi16, Theorem 1.2] is given using background from [DGO17] about hyperbolically embedded subgroups.
- **References:** [Osi16] and [DGO17].

3. Poset Structure of the Families of Hyperbolic and Acylindrically Hyperbolic Structures

- **Speaker:** Jesús Hernández Hernández.
- **Dates:** November 6th and November 13th.
- **Abstract:** This sessions are focused in proving Theorems 2.6, 2.7 and 2.11 of [ABO19]. In Theorem 2.6 the cardinality of the family of acylindrically hyperbolic structures of a group is characterized by means of its geometric actions. In Theorem 2.7 it is shown that the posets of hyperbolic and acylindrically hyperbolic structures of a hyperbolic embedded subgroup always embed in the corresponding posets of the ambient group. In Theorem 2.11 it is verified that the power set of the natural numbers embeds as a poset in the family of acylindrically hyperbolic structures of a acylindrically hyperbolic group.
- **References:** [ABO19].

4. Acylindrically Hyperbolic Accesibility

- **Speaker:** Sandy G. Aguilar Rojas.
- **Date:** November 27th.
- **Abstract:** Dunwoody’s accesibility is roughly discussed in terms of hyperbolic structures. In particular, the notion of \mathcal{AH} -accesibility is introduced following [ABO19, Definition 2.16] and then it is proved that several families of geometric groups are \mathcal{AH} -accesibles following [ABO19, Theorem 2.18].
- **References:** [ABO19].

- Individual Talk:

- **Speaker:** Benachir El Allaoui.
- **Date:** December 4th.

- Individual Talk: “Mackey Functors and Tambara Functors inspired by Equivariant Homotopy Theory”
 - **Speaker:** Ben Spitz.
 - **Date:** December 11th.

References

- [ABO19] Carolyn Abbott, Sahana H. Balasubramanya, and Denis Osin. Hyperbolic structures on groups. *Algebr. Geom. Topol.*, 19(4):1747–1835, 2019.
- [Das23] Sumanta Das. Strong topological rigidity of non-compact orientable surfaces. Preprint at arXiv: 2111.11194 [math.GT], 2023.
- [DGO17] François Dahmani, Vincent Guirardel, and Denis Osin. Hyperbolically embedded subgroups and rotating families in groups acting on hyperbolic spaces. *Mem. Amer. Math. Soc.*, 245(1156):v+152, 2017.
- [dlH79] Pierre de la Harpe. Moyennabilité du groupe unitaire et propriété p de Schwartz des algèbres de von Neumann. In Pierre de la Harpe, editor, *Algèbres d’Opérateurs*, pages 220–227, Berlin, Heidelberg, 1979. Springer Berlin Heidelberg.
- [GdlH15] Rostislav Grigorchuk and Pierre de la Harpe. Amenability and ergodic properties of topological groups: from Bogolyubov onwards. Preprint at arXiv: 1404.7030 [math.GR], 2015.
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- [Mon01] Nicolas Monod. *Continuous bounded cohomology of locally compact groups*, volume 1758 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2001.
- [Mon06] Nicolas Monod. An invitation to bounded cohomology. In *International Congress of Mathematicians. Vol. II*, pages 1183–1211. Eur. Math. Soc., Zürich, 2006.
- [Osi16] Denis Osin. Acylindrically hyperbolic groups. *Trans. Amer. Math. Soc.*, 368(2):851–888, 2016.