# AN EASY PROOF THAT THERE MAY BE NO P-POINTS WITH AN ARBITRARILY LARGE CONTINUUM

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ABSTRACT. We provide an easy proof that there are no P-points in the models obtained by adding many Silver reals with countable support to a model of $\mathsf{CH}$.

## 1. Introduction

The existence of ultrafilters on natural numbers with special combinatorial properties is a major topic in set theory and set-theoretic topology. Ultrafilters with special combinatorial properties are often hard to construct, and more often than not, their existence is independent from the axioms of $\mathsf{ZFC}$. Kunen started this trend in [24], where he proved that there are no Ramsey ultrafilters in the random model. Not long after Shelah found a model with no P-points (see [35, 31]) and Miller showed that there are no Q-points in the Laver model and the Miller model, see [27, 28]. Since then many independence theorems regarding the non-existence of special types of ultrafilters have been proved, see [32, 4, 9, 14, 29, 30, 8, 15, 1, 6, 10] among many others.

It is worth pointing out that although independence results are abundant, there are some $\mathsf{ZFC}$ theorems. Most notably, Kunen's construction of weak P-points in [23]. For some other examples, see [33, 34, 19, 17].

The class of P-points is the one that has received the most attention. This is because of their usefulness, nice combinatorial properties, and good behavior with respect to forcing iteration. Shelah's construction of a model without P-points as presented in [31] and [2] (which is not the same as in [35]) proceeds as follows: for each $\mathsf{P}$-point $\mathcal{U}$, a forcing $\mathbb{P}^\omega(\mathcal{U})$ is defined. This forcing is proper, $\omega^\omega$-bounding and *kills* $\mathcal{U}$; the ultrafilter $\mathcal{U}$ cannot be extended to a $\mathsf{P}$-point in the generic extension obtained by forcing with $\mathbb{P}^\omega(\mathcal{U})$. Moreover, this remains true in any further extension by a proper, $\omega^\omega$-bounding forcing. A model without P-points is then obtained by iterating forcings of type $\mathbb{P}^\omega(\mathcal{U})$, killing each $\mathsf{P}$-point one at a time.[1]

In [14] it was proved that *the Silver forcing* (denoted by $\mathbb{SL}$) kills all ultrafilters in a similar way, although here the preservation property only holds for forcings with the Sacks property. While $\mathbb{P}^\omega(\mathcal{U})$ only kills $\mathcal{U}$, the Silver forcing kills every ultrafilter

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[1] A diamond sequence is needed to guess the names of future P-points.

in the ground model. More precisely, $\mathbb{SL}$ can be decomposed as an iteration of first adding a generic selective ultrafilter $\mathcal{S}$, followed by forcing with $\mathbb{P}(\mathcal{S})$ – a variant of Shelah's poset $\mathbb{P}^{\omega}(\mathcal{S})$. The forcing $\mathbb{P}(\mathcal{S})$ kills all P-points which are not near coherent with $\mathcal{S}$.

It follows that there are no P-points in the Silver model; the model obtained by adding $\omega_2$ Silver reals to a model of $\mathsf{CH}$. This is essentially the only known model without P-points that is obtained with a definable forcing. Furthermore, the argument from [14] also shows that there are no P-points in models obtained by forcing with countable support products of Silver forcing, not just iterations. In particular, the non-existence of P-points is consistent with the continuum being arbitrarily large. The purpose of this note is to present a new proof of this result, which is much easier than the one presented in [14]. Moreover, in our opinion, this is the simplest known proof of non-existence of P-points in any model, regardless of whether the continuum is $\omega_2$ or larger.

The structure of the paper is as follows: After reviewing some notation and preliminaries, in Section 3 we prove two combinatorial results that will be needed in the next section. Both results are easy and well-known, but we include them for completeness. The main result of this paper is presented in Section 4: there are no P-points in any generic extension obtained by forcing (over a model of $\mathsf{CH}$) with a countable support product of Silver forcing of length greater than $\omega_1$. Section 5 contains some thoughts regarding the existence of a non-Tukey top ultrafilter in this type of models.

## 2. Notation and Preliminaries

All ultrafilters are assumed to be non-principal. For $\mathcal{P} \subseteq [\omega]^{\omega}$ and $A \subseteq \omega$, we say that $A$ is *a pseudointersection* of $\mathcal{P}$ if it is almost contained in all elements of $\mathcal{P}$, i.e. $A \setminus P$ is finite for every $P \in \mathcal{P}$. An ultrafilter $\mathcal{U}$ on $\omega$ is a *P-point* if every countable subfamily of $\mathcal{U}$ has a pseudointersection in $\mathcal{U}$. Let $A$ and $B$ be two sets. The expression $f; A \to B$ denotes that $f$ is a partial function from $A$ to $B$. For $p; \omega \to 2$ the domain of $p$ is denoted by $\mathrm{dom}(p)$, and $\mathrm{cod}(p)$ is $\omega \setminus \mathrm{dom}(p)$. For convenience, we will write $p^{-1}(1)$ instead of $p^{-1}[\{1\}]$. The *Silver forcing* (denoted by $\mathbb{SL}$) consists of all partial functions $p; \omega \to 2$ such that $\mathrm{cod}(p)$ is infinite. The order $q \leq p$ is simply $p \subseteq q$. If $G \subseteq \mathbb{SL}$ is a generic filter, the *Silver generic real* is defined as $r_{\mathrm{gen}} = \bigcup G$, which is a total function from $\omega$ to 2.

It is well-known that Silver forcing is proper and $\omega^{\omega}$-bounding (this means that every function in $\omega^{\omega}$ in the generic extension is pointwise bounded by a ground model function). It is easy to see that the set of all $p \in \mathbb{SL}$ such that $p^{-1}(1)$ is infinite is open dense. From now on we assume that every element of $\mathbb{SL}$ has this property. For more on Silver forcing, the reader may consult [20]. For a set of ordinals $X$ we denote by $\mathbb{SL}^{X}$ the set of all functions $p; X \to \mathbb{SL}$ with countable domain, which we denote by $\mathrm{supp}(p)$. Define $q \leq p$ if $\mathrm{supp}(p) \subseteq \mathrm{supp}(q)$ and $q(\alpha) \leq p(\alpha)$ for all $\alpha \in \mathrm{supp}(p)$. It is not hard to prove that $\mathbb{SL}^{X}$ is an $\omega^{\omega}$-bounding proper forcing and $\mathsf{CH}$ implies that it has the $\omega_2$ chain condition, see [3, 22].

## 3. Two Folklore results

Let $\kappa$ be an uncountable cardinal. We say that $\mathcal{S} \subseteq [\kappa]^{\omega}$ is a *stick family for* $\kappa$ if every $A \in [\kappa]^{\kappa}$ contains an element of $\mathcal{S}$. For applications and results on stick families, see e.g. [3, 7, 13, 12, 5, 11]. We are concerned when the ground model remains a stick family in a generic extension. There is a lot of research on this topic, see [18, 16, 26, 21]. The following result is easy to prove.

**Proposition 1** (CH). *Let $\kappa$ be a cardinal. $\mathbb{SL}^\kappa$ forces that $V \cap [\omega_2]^\omega$ is a stick family for $\omega_2$.*

*Proof.* Let $p \in \mathbb{SL}^\kappa$ and $\dot{B}$ such that $p \Vdash \dot{B} \in [\omega_2]^{\omega_2}$. Find $\{(p_\alpha, \beta_\alpha) \mid \alpha \in \omega_2\}$ such that for every $\alpha, \gamma \in \omega_2$

(1) $p_\alpha \leq p$,
(2) $p_\alpha \Vdash \beta_\alpha \in \dot{B}$,
(3) $\beta_\alpha \neq \beta_\gamma$ whenever $\alpha \neq \gamma$.

Since we assume CH, we can apply the $\Delta$-system lemma (Lemma III.6.15 of [25]) to $\{\text{supp}(p_\alpha) \mid \alpha \in \omega_2\} \subseteq [\kappa]^{\leq\omega}$ and find $W \in [\omega_2]^{\omega_2}$ and $R \in [\kappa]^{\leq\omega}$ such that $\{\text{supp}(p_\alpha) \mid \alpha \in W\}$ is a $\Delta$-system lemma with root $R$. Since $\mathbb{SL}^R$ has size at most $\omega_1$ (exactly $\omega_1$ in case $R \neq \emptyset$), there is $Y \in [W]^{\omega_2}$ such that $p_\alpha \restriction R = p_\gamma \restriction R$ for all $\alpha, \gamma \in Y$. Choose $X \subseteq Y$ countable and define $q = \bigcup\limits_{\alpha \in X} p_\alpha$. It follows that $q \Vdash \{\beta_\alpha \mid \alpha \in X\} \subseteq \dot{B}$. □

In fact a stronger statement holds; Baumgartner proved in [3] that $\mathbb{SL}^\kappa$ forces that $V \cap [\omega_1]^\omega$ is a stick family for $\omega_1$. We will use the version for $\omega_2$ since its proof is considerably simpler than the one for $\omega_1$ and sufficient for our purposes.

As in [25], $V^{\mathbb{P}}$ denotes the class of all $\mathbb{P}$-names for a partial order $\mathbb{P}$. Let $\kappa$ be a cardinal, $\dot{\mathcal{U}}$ an $\mathbb{SL}^\kappa$-name for an ultrafilter on $\omega$, and $X \subseteq \kappa$. The *restriction of $\dot{\mathcal{U}}$ to $\mathbb{SL}^X$*, denoted $\dot{\mathcal{U}}_X$, is the $\mathbb{SL}^X$-name for the family

$$\{\dot{A} \in V^{\mathbb{SL}^X} \mid \exists p \in G_X \ p \Vdash \dot{A} \in \dot{\mathcal{U}}\},$$

where $\dot{G}_X$ is the canonical name for the generic filter on $\mathbb{SL}^X$. We will say that $X$ is *$\dot{\mathcal{U}}$-suitable* if $\mathbb{SL}^X$ forces that $\dot{\mathcal{U}}_X$ is an ultrafilter. The following may be considered folklore:

**Lemma 2** (CH). *Let $\kappa$ be a cardinal, $p \in \mathbb{SL}^\kappa$ and $\dot{\mathcal{U}}$ an $\mathbb{SL}^\kappa$-name for an ultrafilter on $\omega$. There is $X \in [\kappa]^{\leq\omega_1}$ that is $\dot{\mathcal{U}}$-suitable.*

*Proof.* Construct $\{M_\alpha \mid \alpha \leq \omega_1\}$ a continuous increasing chain of elementary submodels of $H(\lambda)$ (for some large enough $\lambda$) with the following properties:

(1) $\dot{\mathcal{U}}, \kappa \in M_0$.
(2) If $\alpha < \omega_1$, then $M_\alpha$ is countable.
(3) $[M_{\omega_1}]^\omega \subseteq M_{\omega_1}$.

This is easy to do using CH. Let $X = M_{\omega_1} \cap \kappa$. Note that $\mathbb{SL}^X \subseteq M_{\omega_1}$ and every countable subset of $\mathbb{SL}^X$ is also in $M_{\omega_1}$. We claim that $X$ is $\dot{\mathcal{U}}$-suitable. To see this let $p \in \mathbb{SL}^X$ and $\dot{B}$ be an $\mathbb{SL}^X$-name for a subset of $\omega$. For every $n \in \omega$ find a maximal antichain of conditions $A_n$ in $M_{\omega_1}$ such that every element of $A_n$ decides if $n$ is an element of $\dot{B}$. Since $\mathbb{SL}^X$ is proper, there is $q \in \mathbb{SL}^X$, $q \leq p$ such that each $q \| A_n = \{r \in A_n \mid q \parallel r\}$ is countable.[2] By the previous remarks we can find $\alpha < \omega_1$ such that $q$ and $\{q \| A_n \mid n \in \omega\}$ belong to $M_\alpha$. In particular, $q$ forces that $\dot{B}$ is equivalent to a $\mathbb{SL}^X$-name in $M_\alpha$, so any $(M_\alpha, \mathbb{SL}^X)$-generic filter containing $q$ decides if $\dot{B}$ is in $\dot{\mathcal{U}}$. □

[2] $q \parallel r$ denotes that $q$ and $r$ are compatible.

For our proof in the next section it is only sufficient to know that for every ground model set the membership in $\dot{\mathcal{U}}$ is decided in $V[G_X]$, a fact which is even easier to prove.

## 4. No more P-points

This section contains our main result: Adding at least $\omega_2$ Silver reals with countable support over a model of CH provides a model with no P-points.

For $r;\omega\to 2$ with $r^{-1}(1)$ infinite define the interval partition $P(r)=\{P_n(r)\mid n\in\omega\}$ where $P_n(r)=\{\ell\in\omega\mid |\ell\cap r^{-1}(1)|=n\}$. Interval partitions induced by Silver generic reals are particularly interesting and will be fundamental for our main theorem. Fix a regular cardinal $\kappa>\omega_1$. For $\alpha<\kappa$ and $i<2$ denote by $\dot{r}_\alpha$ the name of the $\alpha$-generic real added by $\mathbb{SL}^\kappa$, and $\dot{D}_i(\alpha)$ is the $\mathbb{SL}^\kappa$-name of $\bigcup\{P_{2m+i}(\dot{r}_\alpha)\mid m\in\omega\}$.

**Proposition 3** (CH)**.** *Let $\dot{\mathcal{U}}$ be an $\mathbb{SL}^\kappa$-name for a nonprincipal ultrafilter on $\omega$, $X\in[\kappa]^{\leq\omega_1}$ an $\dot{\mathcal{U}}$-suitable set and $B\in[\kappa\setminus X]^\omega$. If $G\subseteq\mathbb{SL}^\kappa$ is a generic filter and $i\in 2$, then every pseudointersection of $\{\dot{D}_i(\alpha)\mid\alpha\in B\}$ is disjoint with some element of $\dot{\mathcal{U}}_X[G]$.*

*Proof.* Let $B=\{\alpha_n\mid n\in\omega\}$ and $p\in\mathbb{SL}^\kappa$, we can assume that $B\subseteq\operatorname{supp}(p)$. Let $\dot{Z}$ be a name of a pseudointersection of $\{\dot{D}_i(\alpha_n)\mid n\in\omega\}$. Since $\mathbb{SL}^\kappa$ is $\omega^\omega$-bounding, we may assume there is a ground model increasing function $f\colon\omega\to\omega$ such that $p\Vdash(\dot{Z}\setminus f(n))\subseteq\dot{D}_i(\alpha_n)$. We now find an interval partition $\{E_n\mid n\in\omega\cup\{-1\}\}$ such that the following holds:

1. $f(n)<\min E_{2n}$ for $n\in\omega$.
2. $E_{2n+j}\cap\operatorname{cod}(p(\alpha_n))\neq\emptyset$ for every $n\in\omega$ and $j\in 2$.

Define $U_0=\bigcup\{E_{2n+1}\mid n\in\omega\}$, this is a ground model set. Since $X$ is $\dot{\mathcal{U}}$-suitable and $B\cap X=\emptyset$, we can find $p_1\leq p$, $p\restriction B=p_1\restriction B$ such that $p_1$ decides if $U_0$ is in $\dot{\mathcal{U}}$. If $p_1\Vdash U_0\in\dot{\mathcal{U}}$, let $U=U_0$ and $A_n=E_n$ for $n\in\omega\cup\{-1\}$. In case $p_1\Vdash U_0\notin\dot{\mathcal{U}}$, let $A_{-1}=E_{-1}\cup E_0$ and $A_n=E_{n+1}$ for $n\in\omega$. Define $U=\bigcup\{A_{2n+1}\mid n\in\omega\}$. In either case, we have the following:

1. $\{A_n\mid n\in\omega\cup\{-1\}\}$ is an interval partition.
2. $U=\bigcup\{A_{2n+1}\mid n\in\omega\}$ and $p_1\Vdash U\in\dot{\mathcal{U}}$.
3. $f(n)<\min A_{2n}$ for every $n\in\omega$.
   In particular, $p_1\Vdash\dot{Z}\cap A_{2n+1}\subseteq\dot{D}_i(\alpha_n)$.
4. $A_{2n}\cap\operatorname{cod}(p_1(\alpha_n))\neq\emptyset$ for every $n\in\omega$.

Let $p_2\leq p_1$ be a condition such that for every $n\in\omega$ the following holds:

1. $A_l\cap\operatorname{cod}(p_2(\alpha_n))=\emptyset$ for every $l<2n$.
2. $|A_{2n}\cap\operatorname{cod}(p_2(\alpha_n))|=1$
3. $A_{2n+1}\cap\operatorname{cod}(p_2(\alpha_n))=\emptyset$

Let $a_n$ be the unique element of $A_{2n}\cap\operatorname{cod}(p_2(\alpha_n))$ and define the set

$$H_n=A_{2n+1}\cap\bigcup\{P_{2m+i}(p_2(\alpha_n))\mid m\in\omega\}.$$

It is easy to see that for every $q\leq p_2$ the following hold:

1. If $q(\alpha_n)(a_n)=0$, then $q\Vdash A_{2n+1}\cap\dot{D}_i(\alpha_n)=H_n$.
2. If $q(\alpha_n)(a_n)=1$, then $q\Vdash A_{2n+1}\setminus\dot{D}_i(\alpha_n)=H_n$. In particular, $q\Vdash H_n\cap\dot{D}_i(\alpha_n)=\emptyset$.

Define $H = \bigcup\{H_n \mid n \in \omega\}$, note that $H$ is a ground model set. Once again, find $p_3 \leq p_2$ that decides if $H$ is an element of $\dot{\mathcal{U}}$ and $p_3 \restriction B = p_2 \restriction B$. We will now proceed by cases; first, assume that $p_3 \Vdash H \notin \dot{\mathcal{U}}$. Let $q$ be an extension of $p_3$ such that $q(\alpha_n)(a_n) = 0$ for every $n \in \omega$. We know that $q \Vdash A_{2n+1} \cap \dot{D}_i(\alpha_n) = H_n$ for every $n \in \omega$. Thus, $q$ forces the following:

$$\dot{Z} \cap U \;=\; \bigcup_{n\in\omega} (\dot{Z} \cap A_{2n+1}) \;\subseteq\; \bigcup_{n\in\omega} (\dot{D}_i(\alpha_n) \cap A_{2n+1}) \;=\; \bigcup_{n\in\omega} H_n \;=\; H \;\notin\; \dot{\mathcal{U}}.$$

So, $q \Vdash \dot{Z} \cap (U \setminus H) = \emptyset$. This finishes the proof for this case, since $U \setminus H$ is forced to be in $\dot{\mathcal{U}}$.

We are left with the case $p_3 \Vdash H \in \dot{\mathcal{U}}$. Let $q$ be an extension of $p_3$ such that $q(\alpha)(a_n) = 1$ for every $n \in \omega$. Now $q \Vdash H_n \cap \dot{D}_i(\alpha_n) = \emptyset$ for every $n \in \omega$. It follows that $q$ forces the following:

$$\dot{Z} \cap H = \bigcup_{n\in\omega} (\dot{Z} \cap H_n) \subseteq \bigcup_{n\in\omega} (\dot{D}_i(\alpha_n) \cap A_{2n+1} \cap H_n) = \bigcup_{n\in\omega} (\dot{D}_i(\alpha_n) \cap H_n) = \emptyset.$$

□

The main result now follows easily.

**Theorem 4** (CH)**.** *Let $\kappa > \omega_1$ be a regular cardinal. $\mathbb{SL}^\kappa$ forces that there are no P-points.*

*Proof.* Let $\dot{\mathcal{U}}$ be an $\mathbb{SL}^\kappa$-name for an ultrafilter and $p \in \mathbb{SL}^\kappa$. By Proposition 2 we can find an $\dot{\mathcal{U}}$-suitable set $X \in [\kappa]^{\omega_1}$. For every $\alpha \in \kappa$ let $\dot{e}_\alpha$ be an $\mathbb{SL}^\kappa$-name for which $p \Vdash \dot{D}_{\dot{e}_\alpha}(\alpha) \in \dot{\mathcal{U}}$. Pick $W \in [\kappa]^{\omega_2}$ such that $W \cap X = \emptyset$. By extending $p$ if necessary, we may assume that there is $i \in 2$ such that $p$ forces that $\{\alpha \in W \mid \dot{e}_\alpha = i\}$ has size $\omega_2$. Apply Proposition 1 to find a ground model countable set $B \subseteq W$ and $q \leq p$ such that $q \Vdash \dot{e}_\alpha = i$ for every $\alpha \in B$. Proposition 3 entails that $q$ forces that $\{\dot{D}_i(\alpha) \mid \alpha \in B\}$ is a countable subset of $\dot{\mathcal{U}}$ that does not have a pseudointersection in $\dot{\mathcal{U}}$. □

## 5. Towards Tukey Top

Let $\mathcal{U}$ be an ultrafilter on natural numbers. We say that $\mathcal{U}$ is *Tukey top* if there is $\mathcal{W} \in [\mathcal{U}]^{\mathfrak{c}}$ such that $\bigcap \mathcal{W}_1 \notin \mathcal{U}$ for every $\mathcal{W}_1 \in [\mathcal{W}]^\omega$ (here $\mathfrak{c}$ denotes the cardinality of the set of real numbers). It was a long standing problem of Isbell whether ZFC implies the existence of a non-Tukey top ultrafilter. The problem was recently solved in the negative by Cancino and Zapletal in [10]. However, it is still not known whether a non-Tukey-top ultrafilter exists in models obtained by adding Silver reals, either by iteration or countable support product. The proof of Theorem 4 originated from our attempt to show that there are no Tukey-top ultrafilters in such models. Although we were ultimately unable to prove this result, we believe that we got close. Let us sketch our attempted proof.

Assume CH, let $\kappa$ be a regular uncountable cardinal, and $\dot{\mathcal{U}}$ be an $\mathbb{SL}^\kappa$-name for an ultrafilter. Find $X, i$ and $W$ as in the proof of Theorem 4, only this time make sure $W$ has size $\kappa$. Proposition 3 implies that if $B \in [W]^\omega \cap V$, then $\bigcap\{D_i(\dot{r}_\alpha) \mid \alpha \in B\}$ cannot be in $\dot{\mathcal{U}}$. The problem is that we do not know what to do in case $B \notin V$. We believed that this argument can be refined to take care of all countable subsets of $W$; this would imply that $\dot{\mathcal{U}}$ is forced to be Tukey top.

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## References

[1] Bohuslav Balcar, Ryszard Frankiewicz, and Charles Mills. More on nowhere dense closed $P$-sets. *Bull. Acad. Polon. Sci. Sér. Sci. Math.*, 28(5-6):295–299, 1980.
[2] Tomek Bartoszyński and Haim Judah. *Set theory: on the structure of the real line.* Wellesley, MA: A. K. Peters Ltd., 1995.
[3] James E. Baumgartner. Almost-disjoint sets, the dense set problem and the partition calculus. *Ann. Math. Logic*, 9(4):401–439, 1976.
[4] Jörg Brendle. Between $P$-points and nowhere dense ultrafilters. *Israel J. Math.*, 113:205–230, 1999.
[5] Jörg Brendle. Cardinal invariants of the continuum and combinatorics on uncountable cardinals. *Ann. Pure Appl. Logic*, 144(1-3):43–72, 2006.
[6] Jörg Brendle, Barnabás Farkas, and Jonathan Verner. Towers in filters, cardinal invariants, and Luzin type families. *J. Symb. Log.*, 83(3):1013–1062, 2018.
[7] S. Broverman, J. Ginsburg, K. Kunen, and F. D. Tall. Topologies determined by $\sigma$-ideals on $\omega_1$. *Canad. J. Math.*, 30(6):1306–1312, 1978.
[8] Jonathan Cancino, Osvaldo Guzmán, and Michael Hrušák. Ultrafilters and the Katětov order. *Zbornik Radova*, ?(22(30)):137–176, 2025.
[9] J. Cancino-Manríquez. Every maximal ideal may be Katětov above of all $F_\sigma$ ideals. *Trans. Amer. Math. Soc.*, 375(3):1861–1881, 2022.
[10] Jonathan Cancino-Manríquez and Jindrich Zapletal. On the isbell problem, 2024.
[11] Jorge Antonio Cruz Chapital. $\kappa$-Barely independent families and Tukey types of ultrafilters, 2025.
[12] William Chen. Variations of the stick principle. *Eur. J. Math.*, 3(3):650–658, 2017.
[13] William Chen, Shimon Garti, and Thilo Weinert. Cardinal characteristics of the continuum and partitions. *Israel J. Math.*, 235(1):13–38, 2020.
[14] David Chodounský and Osvaldo Guzmán. There are no P-points in Silver extensions. *Israel J. Math.*, 232(2):759–773, 2019.
[15] Alan Dow. P-filters and Cohen, random, and Laver forcing. *Topology Appl.*, 281:107200, 16, 2020.
[16] Mirna Džamonja and Saharon Shelah. Similar but not the same: various versions of ♣ do not coincide. *J. Symbolic Logic*, 64(1):180–198, 1999.
[17] Jana Flašková. More than a 0-point. *Comment. Math. Univ. Carolin.*, 47(4):617–621, 2006.
[18] Sakaé Fuchino, Saharon Shelah, and Lajos Soukup. Sticks and clubs. *Ann. Pure Appl. Logic*, 90(1-3):57–77, 1997.
[19] Osvaldo Guzmán and Michael Hrušák. On Pospíšil ideals. *Topology Appl.*, 259:242–250, 2019.
[20] Lorenz J. Halbeisen. *Combinatorial set theory.* Springer Monographs in Mathematics. Springer, Cham, 2017. With a gentle introduction to forcing, Second edition of [ MR3025440].
[21] Michal Hrušák. Life in the Sacks model. *Acta Univ. Carolin. Math. Phys.*, 42(2):43–58, 2001. 29th Winter School on Abstract Analysis (Lhota nad Rohanovem/Zahrádky u České Lípy, 2001).
[22] Piotr Koszmider. A formalism for some class of forcing notions. *Z. Math. Logik Grundlag. Math.*, 38(5-6):413–421, 1992.
[23] K. Kunen. Weak $P$-points in $\mathbf{N}^*$. In *Topology, Vol. I, II (Proc. Fourth Colloq., Budapest, 1978)*, volume 23 of *Colloq. Math. Soc. János Bolyai*, pages 741–749. North-Holland, Amsterdam-New York, 1980.
[24] Kenneth Kunen. Some points in $\beta N$. *Math. Proc. Cambridge Philos. Soc.*, 80(3):385–398, 1976.
[25] Kenneth Kunen. *Set theory*, volume 34 of *Studies in Logic (London)*. College Publications, London, 2011.
[26] Heike Mildenberger. Finding generic filters by playing games. *Arch. Math. Logic*, 49(1):91–118, 2010.

[27] Arnold W. Miller. There are no $Q$-points in Laver's model for the Borel conjecture. *Proc. Amer. Math. Soc.*, 78(1):103–106, 1980.

[28] Arnold W. Miller. Rational perfect set forcing. In *Axiomatic set theory (Boulder, Colo., 1983)*, volume 31 of *Contemp. Math.*, pages 143–159. Amer. Math. Soc., Providence, RI, 1984.

[29] Dilip Raghavan and Juris Steprāns. Stable ordered-union versus selective ultrafilters. *Adv. Math.*, 459:Paper No. 109999, 57, 2024.

[30] Dilip Raghavan and Juris Steprāns. Adding ultrafilters to Shelah's model for no nowhere dense ultrafilters, 2024.

[31] Saharon Shelah. *Proper and Improper Forcing*. Perspectives in Mathematical Logic. Springer-Verlag, Berlin, second edition, 1998.

[32] Saharon Shelah. There may be no nowhere dense ultrafilter. In *Logic Colloquium '95 (Haifa)*, volume 11 of *Lecture Notes Logic*, pages 305–324. Springer, Berlin, 1998.

[33] Jan van Mill. An introduction to $\beta\omega$. In *Handbook of set-theoretic topology*, pages 503–567. North-Holland, Amsterdam, 1984.

[34] Jonathan Verner. Lonely points in $\omega^*$. *Topology Appl.*, 155(16):1766–1771, 2008.

[35] Edward L. Wimmers. The Shelah $P$-point independence theorem. *Israel J. Math.*, 43(1):28–48, 1982.

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