The positive formalism

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R. O., Adv. Theor. Math. Phys. **203** (2019) 437–592
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What is the positive formalism?

An axiomatic framework for formulating physical theories.

Accommodates:

- classical statistical mechanics
- the standard formulation of quantum theory
- quantum field theory*
- generalized probabilistic theories
- a timeless formulation of quantum theory*

Should accommodate:

- quantum gravity

What is the positive formalism?

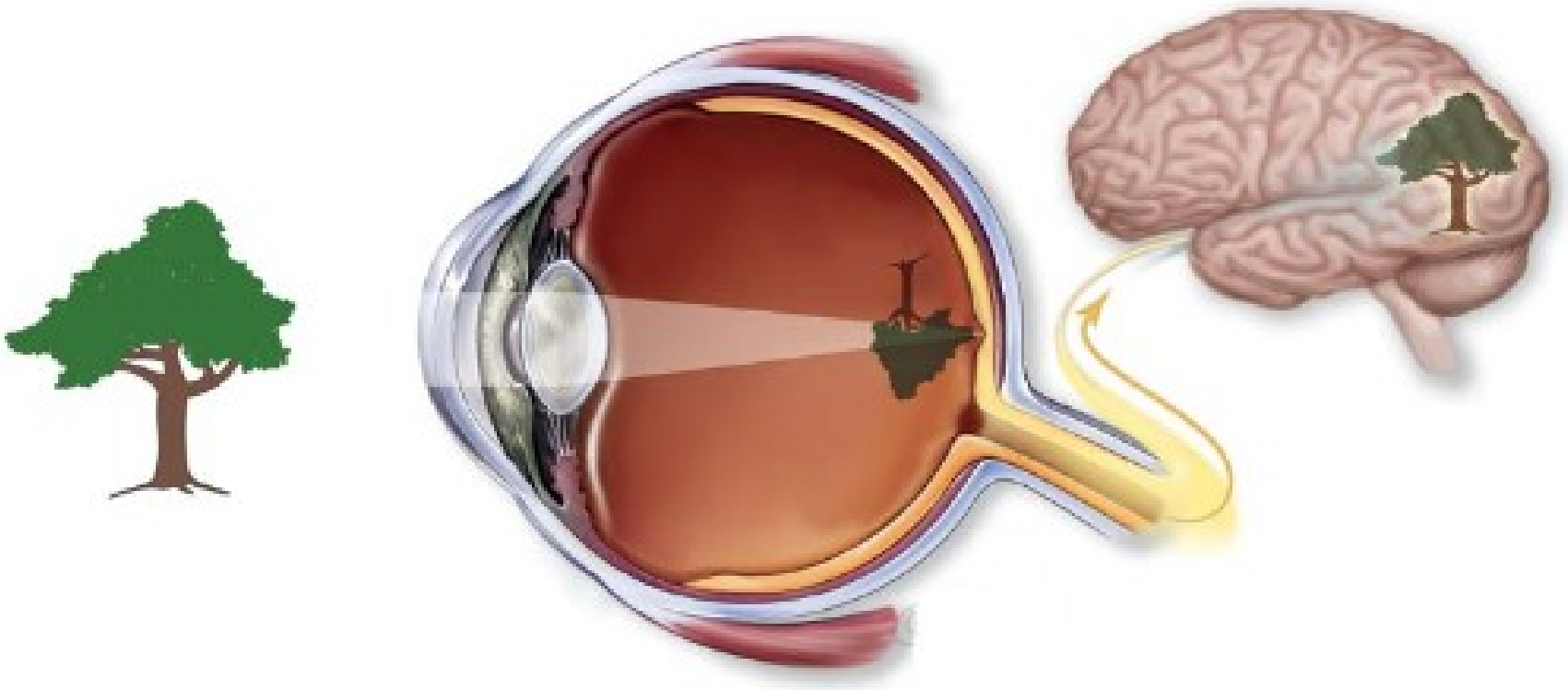
An axiomatic framework for formulating physical theories. **(PF)**

Accommodates:

- classical statistical mechanics **(PF+T+N+C)**
- the standard formulation of quantum theory **(PF+T+N+Q)**
- quantum field theory* **(PF+LOC+Q)**
- generalized probabilistic theories **(PF+T+N)**
- a timeless formulation of quantum theory* **(PF+Q)**

Should accommodate:

- quantum gravity **(PF+LOC+Q) ?**



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Partially ordered vector spaces

Partially ordered vector space

A **real vector space** V with a **partial order** such that:

- $a \leq b \iff a + c \leq b + c \quad \forall a, b, c \in V$ (**translation invariance**)
- $a \leq b \iff \lambda a \leq \lambda b \quad \forall a, b \in V, \forall \lambda > 0$ (**scale invariance**)

Positive map

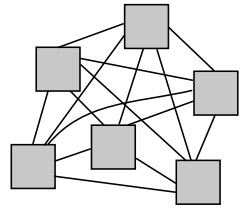
A linear map that maps positive elements to positive elements.

Order unit

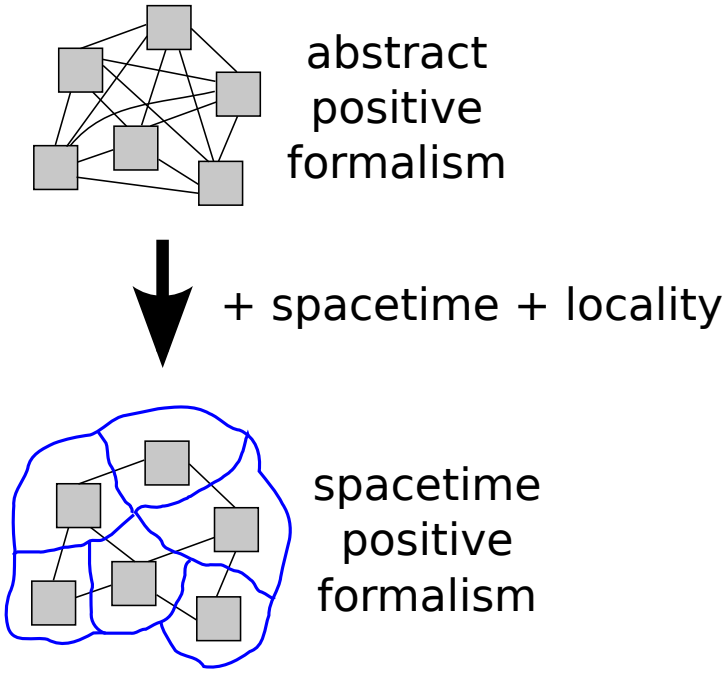
An element $\mathbf{e} \in V$ such that for any $v \in V$ there is $\lambda > 0$ with $v \leq \lambda \mathbf{e}$.

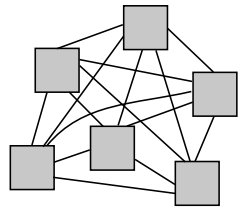
Inner product

A compatible positive-definite **inner product** $(\!|\cdot, \cdot|\!) : V \times V \to \mathbb{R}$.



abstract
positive
formalism


abstract
positive
formalism
+ spacetime + locality
spacetime
positive
formalism



abstract
positive
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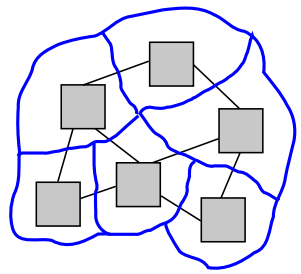
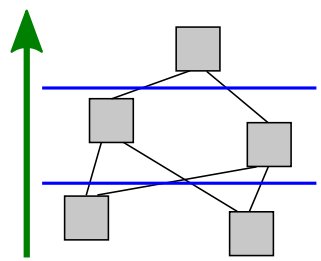
+ spacetime + locality



spacetime
positive
formalism

+ time + causality

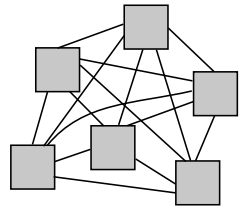


convex
operational
framework

classical
(lattices)

quantum
(anti-lattices)

abstract classical statistical theory

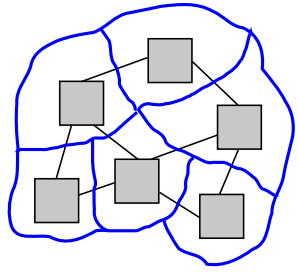


abstract positive formalism

abstract quantum theory

+ spacetime + locality

spacetime statistical field theory

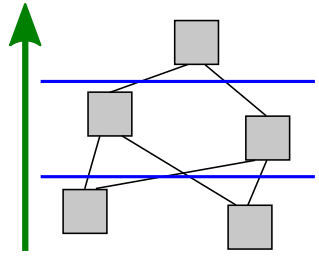


spacetime positive formalism

general boundary quantum field theory

+ time + causality

statistical mechanics



convex operational framework

standard formulation of quantum theory