Quantum Gravity and the Foundations of Quantum Theory

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LQP 34
26 April 2014

Motivation

In the foundations of modern **classical physics**, **time does not play a special role**. When time is singled out in the description of a system, this is merely for convince. In other cases, such as in special or general relativity, it is more convenient to think of time as derived from spacetime. But we can even imagine a classical dynamics in the complete absence of a notion of time.

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In the foundations of modern **classical physics**, **time does not play a special role**. When time is singled out in the description of a system, this is merely for convince. In other cases, such as in special or general relativity, it is more convenient to think of time as derived from spacetime. But we can even imagine a classical dynamics in the complete absence of a notion of time.

Not so in **quantum theory**. A **predetermined notion of time** enters in an essential way in the standard description of the measurement process. The **noncommutativity** at the very heart of quantum theory arises there in the comparison of measurements with different **temporal** order. This makes quantum theory seemingly **inapplicable in a context that lacks a background time**, such as general relativity.

Into the foundations

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Approach this from two sides:

- Examine known quantum physics with a view towards understanding a universal underlying structure, starting with the **known and tested descriptions** (in particular quantum field theory).
- Reason about the **general structure an operational description** of nature could or should have.

Surprisingly, these approaches seem to converge.

Usually I talk about the first one of these approaches, today I shall talk about the second one.

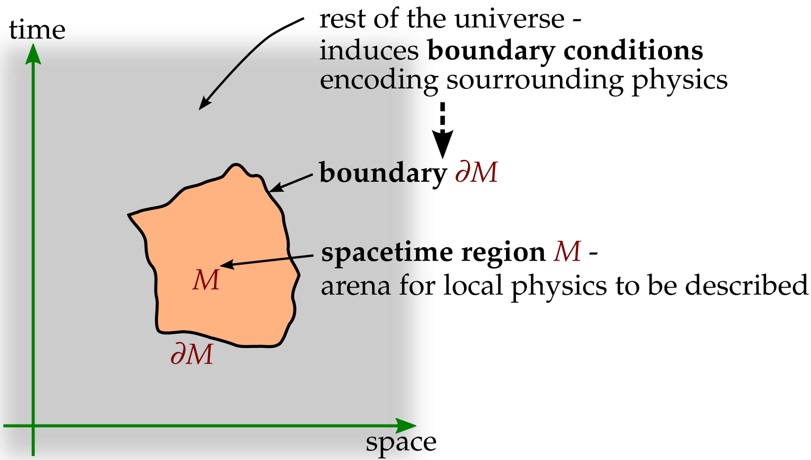
Guidelines

In examine the features of a physical description of nature in the most general terms we shall be guided by two principles:

- **Locality**: We have learned that to understand and describe local physics, a knowledge or control of the immediate spatial and temporal surroundings is sufficient. Details of events far way do not matter for this.
- **Operationalism**: While in classical physics sweeping statements about physical reality in the absence of an observer or actor are possible and even sensible, this is not so in quantum theory. Rather we should be describing physics through the interaction with an observer or experimenter.

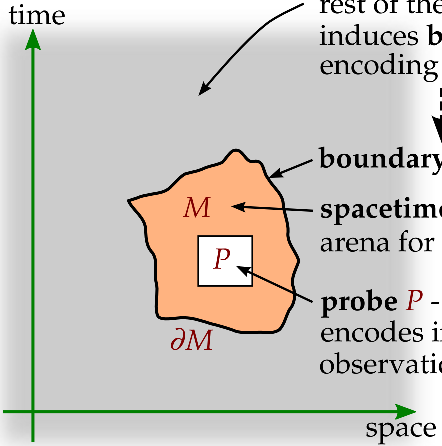
We shall not limit our considerations to quantum physics, but include classical physics as well.

Locality and spacetime



Require a **notion of spacetime**:
spacetime regions and their **boundaries**.

Probes



rest of the universe -
induces **boundary conditions**
encoding sourrounding physics

boundary ∂M

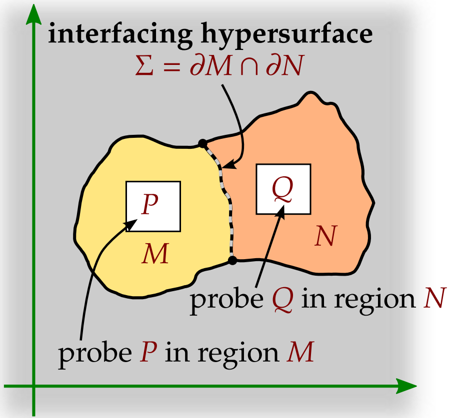
spacetime region M -
arena for local physics to be described

probe P -
encodes interaction with local physics:
observation, preparation, etc.

A **probe** is associated to a spacetime region.
There is also a special **null-probe** representing the absence of a probe.

Composition

For a comprehensive description it is essential that we be able to relate the physics in adjacent spacetime regions.



Need an operation that allows to **combine probes** P, Q in adjacent spacetime regions M, N to a composite probe $P \diamond Q$ in the joint region $M \cup N$.

"Holography"

Information about local physics is communicated between adjacent regions through **boundary conditions** on **interfacing hypersurfaces**.

Towards describing physics

In order to make quantitative descriptions of physical processes we need to **associate suitable mathematical structures** to the ingredients identified so far.

- To a **hypersurface** Σ we associate a space $\mathcal{B}_\Sigma$ of **boundary conditions**. This encodes the possible physical information flows between regions adjacent to the hypersurface. In the special case of boundaries this encodes the influence of the "rest of the universe".
- To a **probe** P in a **spacetime region** M with **boundary condition** $b \in \mathcal{B}_{\partial M}$ we associate a **value** w. This encodes the correlation between boundary conditions, probe, and the physics in the interior.

Values

The values $w(M, P, b)$ might be (among other things!)

- **physical quantities**
  - Truth values: $w \in \{\text{True, False}\}$ indicating e.g. physical realizability or binary outcomes of deterministic experiments
  - Probabilities: $w \in [0, 1]$ indicating e.g. the probability for a binary observation or experimental outcome
  - Expectations: $w \in \mathbb{R}$ indicating e.g. the value of a measured quantity
- **auxiliary quantities**
  - Relative probabilities: $w \in [0, \infty]$
  - Relative expectations: $w \in \mathbb{R}$

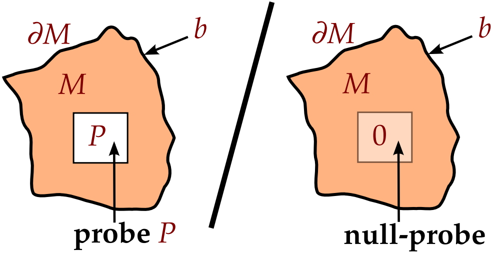
Values might also be "multi-dimensional", e.g., $w \in \mathbb{R}^n$, but we could absorb this in a redefinition of probes.

In general we should expect values only to be auxiliary quantities.

Physical quantities from relative values

If values are only auxiliary, we need to **relate** different probes and or boundary conditions in order to extract physical quantities. The quantities obtained are then **conditional** relations.

Simplest case: Condition on a **boundary condition** b by comparing to the **null-probe**.

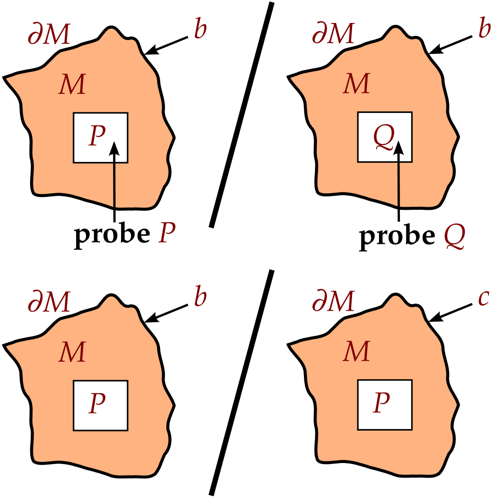


$$\frac{w(M, P, b)}{w(M, 0, b)}$$

is the measurement outcome of probe P in M given boundary condition b

Physical quantities from relative values

Probes and or **boundary conditions** may form **hierarchies** encoded in **partial orderings** that facilitate the extraction of conditional relations.



$$\frac{w(M,P,b)}{w(M,Q,b)}$$

outcome of measurement $P \subset Q$ given apparatus Q in M with boundary conditions b.

$$\frac{w(M,P,b)}{w(M,P,c)}$$

outcome of measurement P for boundary condition $b \subset c$ given that the class of boundary conditions c is present.

Encoding known physics

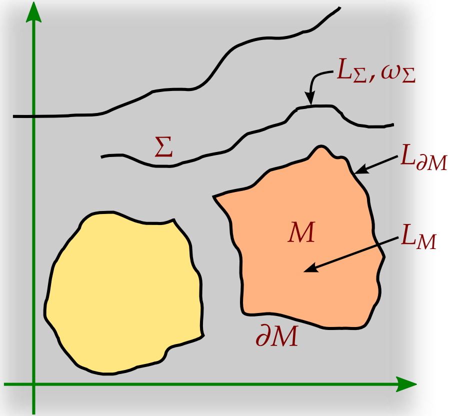
The suitable mathematical structures and their interpretation are distinct in

1. **classical physics**
2. **classical statistical physics**
3. **quantum (statistical) physics**

We consider these in turn.

Classical physics

In classical Lagrangian field theory[1] we are naturally given the following structures:



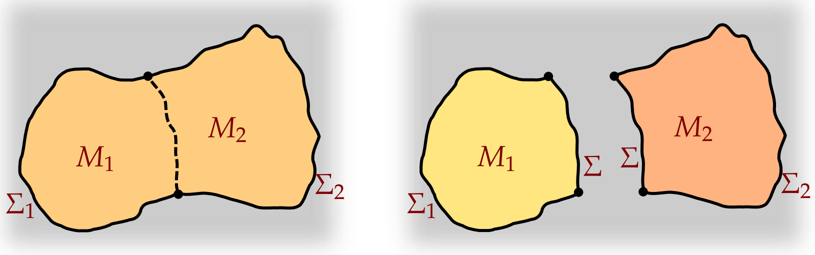
[Kijowski, Tulczyjew 1979], [RO 2010–]

- Per **hypersurface** Σ :
  The **space of solutions** near Σ. This is a **symplectic manifold** $(L_\Sigma, \omega_\Sigma)$.
- Per **region** M :
  The **space of solutions** in M. Forgetting the interior yields a map $L_M \to L_{\partial M}$. Under this map L_M is a **Lagrangian submanifold** $L_M \subseteq L_{\partial M}$.

[1] We consider here the simplest case only, without constraints or gauge symmetries.

Composition of solutions

Consider regions M_1, M_2 with matching boundary components Σ and their **composition** to a joint region $M = M_1 \cup M_2$.



Then we have an **exact sequence**

$$L_M \to L_{M_1} \times L_{M_2} \rightrightarrows L_\Sigma$$

This is a relation between the spaces of solutions in M_1, M_2 and M.

Observables

In classical physics the role of **probes** is taken by **observables**. An observable in a region M is a function $O : L_M \to \mathbb{R}$.

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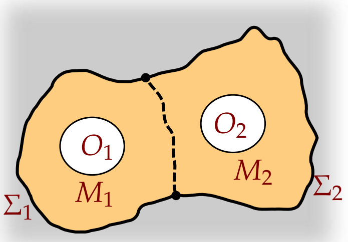
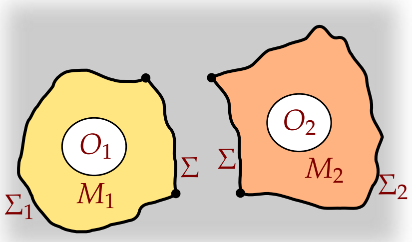




The joint observable $O = O_1 \diamond O_2$ is the product

$$O(\phi) = O(\phi|_{M_1}) \cdot O(\phi|_{M_2})$$

where $\phi \in L_M$.

Physical quantities

- A **boundary condition** on Σ is a boundary solution, i.e., $\mathcal{B}_\Sigma = L_\Sigma$.
- For a spacetime region M and boundary condition $\varphi \in L_{\partial M}$ the value for the **null-probe** is,

$$w(M,0,\varphi) := \begin{cases} 1 & \text{if there is } \phi \in L_M \text{ with } \varphi = \phi|_{\partial M} \\ 0 & \text{otherwise} \end{cases}$$

  This is the truth-value of whether a given boundary condition can be physically realized or not.
- For a spacetime region M a **probe** is an **observable** O in M. To the boundary condition φ assign the value,

$$w(M,O,\varphi) := \begin{cases} O(\phi) & \text{if there is } \phi \in L_M \text{ with } \varphi = \phi|_{\partial M} \\ 0 & \text{otherwise} \end{cases}$$

  If the boundary condition is physically realizable this yields the value of the observable.

Statistical classical physics

- We consider **boundary conditions** that are **probability densities** μ on the space $L_{\partial M}$ of boundary solutions (which may be thought of as **statistical ensembles**).
- As before, **probes** are **observables**. Given an observable O in the spacetime region M with boundary condition μ we define the associated value as,

$$w(M, O, \mu) := \int_{L_M} O(\phi)\, \mu(\phi|_{\partial M})$$

Examples of physical quantities:

$$w(M, 0, \mu)$$

is the fraction of the boundary probability distribution μ that is physically realizable.

$$\frac{w(M, O, \mu)}{w(M, 0, \mu)}$$

is the expectation value of O given the probability distribution induced by the boundary condition μ.

Statistical classical physics – technical remarks

In **statistical mechanics** the symplectic structure yields a natural volume form on L_Σ. This can be used to make sense of the integrals.

The case of **statistical field theory** has not been worked out, however this could be a promising route towards the longstanding problem of a **statistical treatment** of the **general theory of relativity**.

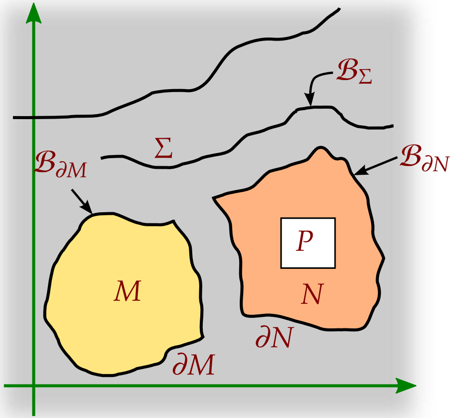
- There are technical challenges concerning measure theory in infinite dimensional spaces.
- It is likely necessary to represent observables as densities and boundary conditions as functions rather than the other way round.

A probabilistic setting

Consider a setting where values are relative and give rise to **probabilities** and **real expectation values**. (Just like in the classical statistical setting.)

- The spaces $\mathcal{B}_\Sigma$ of boundary conditions are **real vector spaces with a partial order**.
- A class of **basic probes** (including the null-probe) on M give rise to **values** that are **positive linear functions** on $\mathcal{B}_{\partial M}$. (This is required for relative probabilities.)
- All **probes** on M give rise to **values** that are **real linear functions** on $\mathcal{B}_{\partial M}$. The space of probes on M itself is a **real vector space with a partial order**.

Spacetime assignments



To the geometric structures associate the data,

- per **hypersurface** Σ : an **ordered vector space** $\mathcal{B}_\Sigma$,
- per **region** M : a **positive map** $w(M,0,\cdot):\mathcal{B}_{\partial M}\to\mathbb{R}$,
- per **region** M that contains a **probe** P : a **real linear map** $w(M,P,\cdot):\mathcal{B}_{\partial M}\to\mathbb{R}$.

Physical quantities

- Given boundary conditions $b \leq c \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{w(M,0,b)}{w(M,0,c)}$$

  is the **conditional probability** for b to be realized given c.

- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition c is given by,

$$\frac{w(M,P,c)}{w(M,0,c)}.$$

Physical quantities – of quantum theory!

- **Boundary conditions** generalize **mixed states** and **projection operators**.
- **Probes** generalize **observables** and **weighted quantum operations**.
- Given boundary conditions $b \leq c \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{w(M,0,b)}{w(M,0,c)}$$

is the **conditional probability** for b to be realized given c. **Transition amplitudes** arise as a special cases of this.
- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition c is given by,

$$\frac{w(M,P,c)}{w(M,0,c)}.$$

Conventional **expectation values** arise as special cases of this.

Quantum theory

It turns out that a formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

Quantum theory

It turns out that a formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

This is called the **general boundary formulation of quantum theory**.

The key point is that the extraction and coherent interpretation of physical quantities in this formulation does not require any notion of time. (But it does require a weak notion of spacetime.)

This suggests a suitable basis for implementing quantum theories in a generally covariant setting.

General boundary formulation

So far, there exist two versions of this:

- The **amplitude formalism**: generalizes Hilbert spaces, amplitudes, observables
  - based on the mathematical framework of **topological quantum field theory** (TQFT) [Witten, Segal, Atiyah, . . . 1988–]
  - can be equipped with present physical interpretation [RO 2005]
- The **positive formalism**: generalizes spaces of mixed states, super operators, quantum operations
  - arises as "modulus square" of amplitude formalism, leads to "positive TQFT" [RO 2012]

The formulation we have arrived at can be identified precisely with the **positive formalism**.

Some applications of the GBF

- Conceptual basis for **spin foam approach** to quantum gravity (sometimes secretly so)
- Non-linear models:
  - **Three dimensional quantum gravity** is a TQFT and fits “automatically”. [Witten 1988;. . .]
  - **Quantum Yang-Mills theory** in 2 dimensions for arbitrary regions and hypersurfaces with corners. [RO 2006]
  - **Yang-Mills theory in higher dimensions** is under investigation [Díaz 2014]
- New **S-matrix** type asymptotic amplitudes [Colosi, RO 2008; Colosi 2009; Dohse 2011; 2012]
- QFT in **curved spacetime**: dS, AdS and more [Colosi, Dohse 2009–]
- Rigorous and functorial quantization of linear and affine field theories without metric background. [RO 2010; 2011; 2012]
- **Unruh effect**. [Colosi, Rätzel 2012; Bianchi, Haggard, Rovelli 2013]
- Striking results for **fermions**: Hilbert spaces become **Krein spaces** and an **emergent notion of time**. [RO 2012]