Preliminaries for Asymptotic Safety

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Notation

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- The classical fields are labeled with a c

$$\phi \rightarrow \phi_c.$$

Effective Field Theories

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- Coarse Graining Procedure.
- The Renormalization Group.
- Exact Renormalization Group Equation.

Coarse Graining Procedure

Consider, for simplicity the scalar case. We want to calculate the spectation values of the observables of the theory, let say

$$\langle \mathcal{O} \rangle = \int \Pi_k \mathcal{D}\phi_k \mathcal{O}(\phi) e^{-S} \tag{1}$$

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$$\langle \mathcal{O} \rangle = \int \Pi_k \mathcal{D}\phi_k \mathcal{O}(\phi) e^{-S} \tag{1}$$

This quantities are infinite. Renormalization is the solution. Consider a cutoff Λ, then (1) becomes

$$\langle \mathcal{O} \rangle = \int \Pi_{k \leq \Lambda} \mathcal{D}\phi_k \mathcal{O}_\Lambda(\phi) e^{-S_\Lambda} \tag{2}$$

The degrees of freedom with momentum greater than Λ are integrated out.

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$$\int \Pi_{k\leq\Lambda}\mathcal{D}\phi_k \mathcal{O}_\Lambda(\phi)e^{-S_\Lambda} = \int \Pi_{k\leq\Lambda-\delta\Lambda}\mathcal{D}\phi_k \mathcal{O}_{\Lambda-\delta\Lambda}(\phi)e^{-S_{\Lambda-\delta\Lambda}} \qquad (3)$$

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$$\int \Pi_{k\leq\Lambda}\mathcal{D}\phi_k\mathcal{O}_\Lambda(\phi)e^{-S_\Lambda} = \int \Pi_{k\leq\Lambda-\delta\Lambda}\mathcal{D}\phi_k\mathcal{O}_{\Lambda-\delta\Lambda}(\phi)e^{-S_{\Lambda-\delta\Lambda}} \tag{3}$$

Therefore, for each change in the cutoff, the explit form of the action and the observable changes. If we fix

$$\langle\mathcal{O}\rangle = 1 \tag{4}$$

from some experiment, then we can know how the action and the observable are in a lower momentum scale. This procedure is called the *Coarse Graining*.

In general, we don't know how the action is going to change.

The Effective Action

We define the genarating functional of the connected Green's functions as

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and the effective action

$$\Gamma = W - \int d^d x \phi_c(x) J(x) \tag{7}$$

which is the generating functional of the 1PI Green's functions.

Momentum Dependence

We want to integrate out degrees of freedom with momentum greater than a cuttof Λ. For notation the cutoff from now is k. Therefore our effective action depend on k

$$\Gamma \longrightarrow \Gamma_k \tag{8}$$

The way in which the action change with the scale is governed by the renormalization group. If we want a general expression for the action at any sacale, we have to consider an action, in principle, with infinite coupling constants.

Renormalization Group

Consider an effective field theory with a configuration space $\mathcal{F}$. Let $\mathcal{Q}$ an infinite smooth manifold, with local coordinates (g_i) and $\mathbb{R}^+$ the positive real line parametrizing the momentum scale. The effective action is defined as

$$\Gamma : \mathcal{F} \times \mathcal{Q} \times \mathbb{R}^+ \longrightarrow \mathbb{C} \tag{9}$$

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In the scalar case

$$\Gamma[\phi, g_i, k] := \Gamma_k[\phi, g_i]$$

Remark: $\mathcal{F} \times \mathcal{Q} \times \mathbb{R}^+$ is the space of all theories, the Theory Space.

What means infinite coupling constants?

Infinite coupling constants, is just a way to write the most general effective action at any scale, for exameple if we have

$$\mathcal{L}_k = \frac{Z_\phi}{2}(\partial\phi)^2 + \frac{1}{2}g_1^2\phi^2 \tag{10}$$

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hence, we can think in a general expresion like

$$\mathcal{L}_k = \frac{Z_\phi}{2}(\partial\phi)^2 + \sum_{i=1}^{\infty} g_i(k)\phi^i \tag{12}$$

Which are the essential coupling constants?

A natural question: These infinite coupling constants are independent of field redefinitions? The answer is not. The coupling constants is split in two sets, one for these which can be absorbed by a field redefinition, the other for its complement. These are called inessential and essential.

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$$\phi \to \phi'(\phi)$$

For this transformation we can find new coupling constants g_i' such that

$$\Gamma_k[\phi'(\phi), g_i] = \Gamma_k[\phi, g_i'] \tag{13}$$

This is a local action of $\mathcal{G}$ on $\mathcal{Q}$.

We split the coupling constants

$$\{g_i\} = \{g_{\hat{\imath}}\} \cup \{g_{\bar{\imath}}\}$$

where $\{g_{\bar{\imath}}\}$ are invariant under $\mathcal{G}$, and $\{g_{\hat{\imath}}\}$ transform non trivially. We said that $\{g_{\hat{\imath}}\}$ and $\{g_{\bar{\imath}}\}$ are local coordinates of the stable and unstable manifold respectivelly.

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We also have invarianza under rescaling, i.e.

$$\Gamma_{bk}[b^{d_\phi}(\phi), b^{d_i} g_i] = \Gamma_k[\phi, g_i] \tag{14}$$

where $b \in \mathbb{R}^+$, d_ϕ and d_i are the canonical dimension of ϕ and g_i.

Expansion of the Effective Action

The most general form of the effective action is

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If we want to know how is the behaviour of the theory at any scale, we have to solve for infinite beta functions.

ERGE

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where

$$\Delta_k S = \frac{1}{2} \int \frac{d^d x}{(2\pi)^d} \mathcal{R}_k(p^2) |\phi(p)|^2 = \frac{1}{2} \int d^d x \phi(x) \mathcal{R}_k(-\partial_x^2) \phi(x) \quad (20)$$

is the cutoff supressing the IR modes with $p^2 << k^2$.

We are just interested in the asymptotic behaviour of $\mathcal{R}_k(p^2)$,

$$\mathcal{R}_k(p^2) = \begin{cases} k^2 & p^2 << k^2 \\ 0 & p^2 >> k^2 \end{cases}$$

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$$\begin{aligned} \mathcal{R}_k(p^2) &= (k^2 - p^2)\theta(k^2 - p^2) \\ \mathcal{R}_k(p^2) &= \frac{p^2}{e^{p^2/k^2} - 1} \end{aligned}$$

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4. The modification of the connected Green's functions is in the mass of the propagator.

Connected Green's Functions

In the Klein-Gordon case

$$S + \Delta_k S = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \left(p^2 + m^2 + \mathcal{R}_k(p^2)\right) |\phi(p)|^2 + \ldots$$

Connected Green's Functions

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therefore the new propagator is

$$\frac{i}{p^2 + m^2 + \mathcal{R}_k(p^2)}$$

Hence W_k is the generating of the C.G.F.

Asymptotic action

We have two asymptotics limits for the action,

$$\lim_{k \to \infty} \Gamma_k , \quad \lim_{k \to 0} \Gamma_k \tag{21}$$

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$$\lim_{k \to \infty} \Gamma_k, \quad \lim_{k \to 0} \Gamma_k \tag{21}$$

to know what these limits are, is convinient to write

$$e^{-\Gamma_k} = \int \mathcal{D}\phi e^{-S + \int d^d x (\phi - \phi_c) \frac{\delta \Gamma_k}{\delta \phi_c}} e^{-\int d^d x (\phi - \phi_c) \mathcal{R}_k (\phi - \phi_c)} \tag{22}$$

If $k \longrightarrow 0$, then $R_k(p^2) \longrightarrow 0$, hence

Effective Action

$$\begin{aligned}\lim_{k\to 0}\Gamma_k &= \lim_{k\to 0}\left(W_k - \int d^dx\phi_c(x)J(x) - \Delta_k S\right)\\ &= W - \int d^dx\phi_c(x)J(x) = \Gamma\end{aligned}$$

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for $k \longrightarrow \infty$ we have $R_k(p^2) \longrightarrow k^2$, then

$$\begin{aligned} e^{-\int d^d x(\phi-\phi_c)\mathcal{R}_k(\phi-\phi_c)} &\longrightarrow e^{-k^2 \int d^d x(\phi-\phi_c)^2}\\ &\longrightarrow \delta(\phi-\phi_c)\end{aligned}$$

then we get from (22)

Bare Action

$$
\begin{aligned}
\lim_{k\to\infty} e^{-\Gamma_k} &\to \int \mathcal{D}\phi\,\delta(\phi-\phi_c)\,e^{-S+\int d^dx(\phi-\phi_c)\frac{\delta\Gamma_k}{\delta\phi_c}} \\
&\to e^{-S} \\
\lim_{k\to\infty} \Gamma_k &\to S
\end{aligned}
$$

The action between these two limits is given by

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the left hand side of this equation satisfy

Exact Renormalization Group Equation

$$k\frac{\partial \Gamma_k}{\partial k} = \frac{1}{2} Tr\left[\left(\Gamma_k^2 + \mathcal{R}_k\right)^{-1} k\frac{\partial \mathcal{R}_k}{\partial k}\right] \tag{24}$$

where Γ_k^2 has the matrix elements

$$\Gamma_k^2(x,y) = \frac{\delta^2 \Gamma_k}{\delta\phi(x)\delta\phi(y)}$$

and $\mathcal{R}_k$ has the matrix elements

$$\mathcal{R}_k(x, y) = \mathcal{R}_k(-\partial_x^2)\delta(x - y)$$

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and the trace of some function (matrix) $F(x,y)$ is defined

$$Tr[F] = \int d^d x F(x,x)$$

The equation which we want to solve to obtain the flow of the essential beta functions $\{\beta_{\bar{\imath}}\}$ is

$$\frac{1}{2} Tr \left[\left(\Gamma_k^2 + \mathcal{R}_k \right)^{-1} k \frac{\partial \mathcal{R}_k}{\partial k} \right] = \sum_i \beta_i(k) \mathcal{P}_i(\phi) \tag{25}$$

Derivation of the ERGE

$$
\begin{aligned}
k\frac{\partial\tilde{\Gamma}_k}{\partial k} &= k\frac{\partial W_k}{\partial k} \\
&= \frac{k}{2e^{-W_k}}\int \mathcal{D}\phi e^{-S-\Delta_k S+\int d^dx\phi J} \times \\
&\quad \int d^dxd^dy\phi(x)\phi(y)\frac{\partial\mathcal{R}_k(-\partial_x^2)}{\partial k}\delta(x-y) \\
&= \frac{1}{2}\int d^dxd^dy\langle\phi(x)\phi(y)\rangle k\frac{\partial\mathcal{R}_k(x,y)}{\partial k}
\end{aligned}
$$

where

$$
\langle\mathcal{N}\rangle = \frac{1}{e^{-W_k}}\int \mathcal{D}\phi\mathcal{N}e^{-S-\Delta_k S+\int d^dx\phi J}
$$

The two-point Green function is

$$
\begin{aligned}
G_k(x,y) &= \frac{\delta^2 W_k[J]}{\delta J(x)\delta J(y)} \\
&= \langle\phi(x)\phi(y)\rangle - \langle\phi(x)\rangle\langle\phi(y)\rangle
\end{aligned}
$$

with this

$$
\begin{aligned}
k\frac{\partial\tilde{\Gamma}_k}{\partial k} &= \frac{1}{2}\int d^dx d^dy G_k(x,y) k\frac{\partial\mathcal{R}_k(x,y)}{\partial k} + \\
&\quad \frac{1}{2}\int d^dx d^dy \langle\phi(x)\rangle\langle\phi(y)\rangle k\frac{\partial\mathcal{R}_k(x,y)}{\partial k} \\
&= \frac{1}{2}\int d^dx G_k(x,x) k\frac{\partial\mathcal{R}_k(-\partial_x^2)}{\partial k} + \\
&\quad \frac{1}{2}\int d^dx \phi_c(x) k\frac{\partial\mathcal{R}_k(-\partial_x^2)}{\partial k}\phi_c(y)
\end{aligned}
$$

$$k\frac{\partial\tilde{\Gamma}_k}{\partial k} = \frac{1}{2}Tr\left[G_k k\frac{\partial\mathcal{R}_k}{\partial k}\right] + \frac{1}{2}\int d^dx\phi_c(x)k\frac{\partial\mathcal{R}_k(-\partial_x^2)}{\partial k}\phi_c(x)$$

hence

$$k\frac{\partial\Gamma_k}{\partial k} = \frac{1}{2}Tr\left[G_k k\frac{\partial\mathcal{R}_k}{\partial k}\right] \tag{26}$$

But

$$\int dzG_k(x,z)\tilde{\Gamma}_k^2(z,y) = \delta(x,y)$$

then

$$G_k = \left(\Gamma_k^2 + \mathcal{R}_k\right)^{-1}$$

finally we get

$$k\frac{\partial \Gamma_k}{\partial k} = \frac{1}{2} Tr\left[\left(\Gamma_k^2 + \mathcal{R}_k\right)^{-1} k\frac{\partial \mathcal{R}_k}{\partial k}\right]$$

Scalar Theory

Consider an effective action

$$\Gamma_k = \int d^d x \left(\frac{1}{2}(\partial\phi)^2 + V_k[\phi^2] \right) \tag{27}$$

then

$$\Gamma_k^2(x,y) = \left(-\partial_x^2 + 2V_k'[\phi^2] + 4\phi^2 V_k''[\phi^2]\right)\delta(x-y)$$

With a function $P(x) = x + \mathcal{R}(x)$ we have

$$\Gamma_k^2 + \mathcal{R}_k = P + 2V_k' + 4\phi^2 V_k''$$

with $t = \log(k/k_0)$ for some fixed k_0

ERGE

$$\partial_t \Gamma_k = \frac{1}{2} Tr \left(\frac{\partial_t P_k}{P_k + 2V_k' + 4\phi^2 V_k''} \right)$$

In general, for a function $W(-\partial^2)$ we have

$$Tr(W(-\partial^2)) = \int d^d x \int \frac{d^d p}{(2\pi)^d} W(p^2) e^{p(x-y)}\Big|_{x=y}$$

In general, for a function $W(-\partial^2)$ we have

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&= \int d^d x \int \frac{d^d p}{(2\pi)^d} W(p^2) \\
&= \int d^d x \int \frac{d\Omega dp_r}{(2\pi)^d} p_r^{d-1} W(p_r^2)
\end{aligned}
$$

with

$$\int d\Omega = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

then we have

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where

$$\begin{aligned} A_d &= \frac{1}{(4\pi)^d} \\ Q_n(W) &= \frac{1}{\Gamma(n)} \int_o^\infty dz z^{n-1} W(z) \end{aligned}$$

The ERGE equation now is

$$\partial_t V_k = \frac{1}{2} A_d Q_{d/2} \left(\frac{\partial_t P_k}{P_k + 2V_k' + 4\phi^2 V_k''} \right) \tag{29}$$

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$$V_k(\phi^2) = \sum_{n=1}^{\infty} \lambda_{2n} \phi^{2n} \tag{30}$$

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where

$$\lambda_{2n} = \frac{1}{n!} \frac{\partial^n V_k}{\partial(\phi^2)^n} \bigg|_{\phi^2=0}$$

As a first calculation, consider a truncation of the serie. Just the first two terms. The beta function equations are

$$\beta_2 = -6\lambda_4 A_d Q_{d/2}\left(\frac{\partial_t P_k}{(P_k+2\lambda_2)^2}\right) \tag{31}$$

$$\beta_4 = 72\lambda_4^2 A_d Q_{d/2}\left(\frac{\partial_t P_k}{(P_k+2\lambda_2)^3}\right) \tag{32}$$

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In order to look the Wilson-Fisher fixed point, we put $d = 3$. With $\tilde{\lambda}_2 = k^{-2}\lambda_2$ and $\tilde{\lambda}_4 = k^{-1}\lambda_4$, we obtain

$$\tilde{\beta}_2 = -2\tilde{\lambda}_2 - 6\tilde{\lambda}_4 k^{-1} A_3 Q_{3/2}\left(\frac{\partial_t P_k}{(P_k+2\lambda_2)^2}\right)$$

$$\tilde{\beta}_4 = -\tilde{\lambda}_4 + 72\tilde{\lambda}_4^2 k A_3 Q_{3/2}\left(\frac{\partial_t P_k}{(P_k+2\lambda_2)^3}\right)$$

Now, with $R_k(z) = (k^2 - z)\theta(k^2 - z)$ we have

$$Q_n\left(\frac{\partial_t P_k}{(P_k + a)^l}\right) = \frac{2}{n!}\frac{1}{(1+\tilde{a})^l}k^{2(n-l+1)}$$

with $\tilde{a} = k^{-2}a$.

Now, with $R_k(z) = (k^2 - z)\theta(k^2 - z)$ we have

$$Q_n\left(\frac{\partial_t P_k}{(P_k + a)^l}\right) = \frac{2}{n!}\frac{1}{(1+\tilde{a})^l}k^{2(n-l+1)}$$

with $\tilde{a} = k^{-2}a$.Then

$$\begin{aligned}
\partial_t\tilde{\lambda}_2 &= -2\tilde{\lambda}_2 - \frac{1}{\pi^2}\frac{2\tilde{\lambda}_4}{(1+2\tilde{\lambda}_2)^2} \\
\partial_t\tilde{\lambda}_4 &= -\tilde{\lambda}_4 + \frac{1}{\pi^2}\frac{24\tilde{\lambda}_4^2}{(1+2\tilde{\lambda}_2)^3}
\end{aligned}$$

The non trivial fixed point is given by

Fixed Point

$$\tilde{\lambda}_2 = -\frac{1}{26} \quad (33)$$

$$\tilde{\lambda}_4 = \frac{72\pi^2}{(13)^3} \quad (34)$$

THANKS

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