The Unruh effect in general boundary QFT

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Morelia, 13 June 2018

Contents

Unruh effect

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Importance:

- Relation between the Minkowski vacuum and the notion of particle in Rindler space (naturally associated with an accelerated observer): particle content of a field theory is observer dependent
- Relation with the Hawking effect and cosmological horizons
- Possible experimental detection

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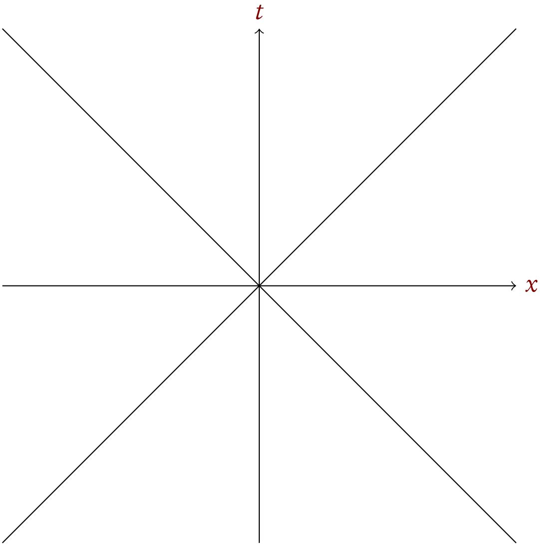
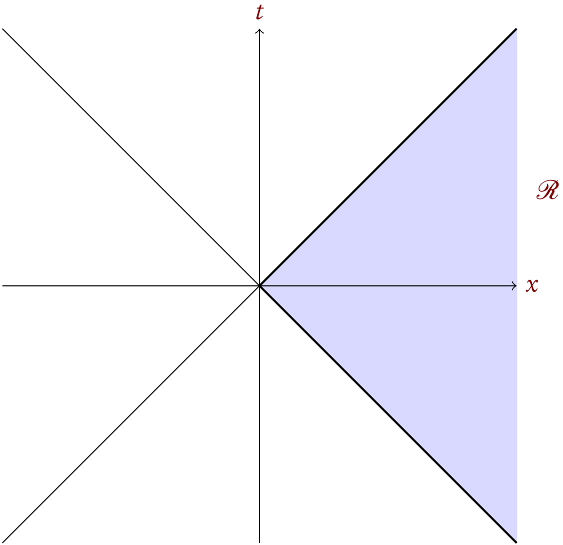
Moreover:

- Perfect arena for the GBF

2d Minkowski



$2d$ Minkowski, $2d$ Rindler



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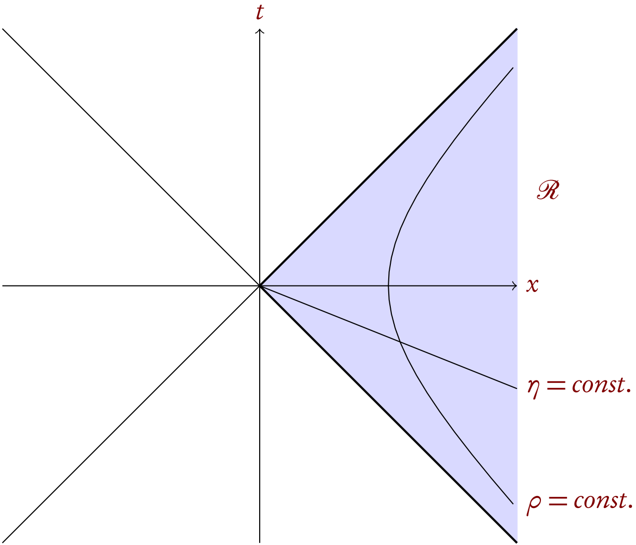

- **Operational interpretation**: A uniformly accelerated Unruh-DeWitt detector responds as if submersed in a thermal bath when interacting with a quantum field in the Minkowski vacuum state.

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- **Particle interpretation**: the vacuum state in Minkowski corresponds to an entangled state between the modes of the field defined in the left and right Rindler wedges.
  - Crispino et al., *The Unruh effect and its applications*, Rev. Mod. Phys. **80** (2008), 787–838
    «the Unruh effect is the equivalence between the Minkowski vacuum and a thermal bath of Rindler particles»

Outline

GBF and QFT

Standard QFT can be formulated within the GBF

GBF and QFT

Standard QFT can be formulated within the GBF

2 quantization schemes have been studied, that transform a classical field theory into a general boundary quantum field theory:

- Schrödinger-Feynman quantization
- holomorphic quantization

Schrödinger-Feynman quantization

- Schrödinger representation + Feynman path integral quantization
  The state space $\mathcal{H}_\Sigma$ for a hypersurface Σ is the space of functions on field configurations K_Σ on Σ.
- Inner product,
$$\langle\psi_2|\psi_1\rangle = \int_{K_\Sigma} \mathcal{D}\varphi\, \psi_1(\varphi)\overline{\psi_2(\varphi)}.$$
- Amplitude for a region M, $\psi \in \mathcal{H}_{\partial M}$,
$$\rho_M(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi\, \psi(\varphi) \int_{K_M, \phi|_{\partial M}=\varphi} \mathcal{D}\phi\, e^{iS_M(\phi)}.$$
- A classical observable F in M is modeled as a function on K_M. The quantization of F is the linear map $\rho_M^F : \mathcal{H}_{\partial M} \to \mathbb{C}$ defined as
$$\rho_M^F(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi\, \psi(\varphi) \int_{K_M, \phi|_{\partial M}=\varphi} \mathcal{D}\phi\, F(\phi) e^{iS_M(\phi)}.$$

Holomorphic quantization

- Linear field theory: L_Σ is the vector space of solutions near the hypersurface Σ.
- L_Σ carries a non-degenerate symplectic structure ω_Σ and a complex structure $J_\Sigma : L_\Sigma \to L_\Sigma$ compatible with the symplectic structure:

$$J_\Sigma^2 = -\mathrm{id}_\Sigma \qquad \text{and} \qquad \omega_\Sigma(J_\Sigma(\cdot), J_\Sigma(\cdot)) = \omega_\Sigma(\cdot,\cdot).$$

- J_Σ and ω_Σ combine to a real inner product $g_\Sigma(\cdot,\cdot) = 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$ and to a complex inner product $\{\cdot,\cdot\}_\Sigma = g_\Sigma(\cdot,\cdot) + 2\mathrm{i}\omega_\Sigma(\cdot,\cdot)$ which makes L_Σ into a complex Hilbert space.
- The Hilbert space $\mathcal{H}_\Sigma$ associated with Σ is the space of holomorphic functions on L_Σ with the inner product

$$\langle \psi, \psi' \rangle_\Sigma = \int_{L_\Sigma} \overline{\psi(\phi)} \psi'(\phi) \exp\left(-\frac{1}{2} g_\Sigma(\phi,\phi)\right) \mathrm{d}\mu(\phi),$$

where μ is a (fictitious) translation-invariant measure on L_Σ.

Holomorphic quantization (II)

- The amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$ associated with the spacetime region M for a state $\psi \in \mathcal{H}_{\partial M}$ is given by

$$\rho_M(\psi) = \int_{L_\Sigma} \psi(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

- The observable map associated to a classical observable F in a region M is

$$\rho_M^F(\psi) = \int_{L_\Sigma} \psi(\phi) F(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

Klein-Gordon theory in Minkowski

- Action of a real massive Klein-Gordon field on $1+1$-dimensional Minkowski spacetime

$$S[\phi] = \frac{1}{2}\int \mathrm{d}^2x \left(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right).$$

- The GBF is defined in a region M bounded by the disjoint union of two spacelike hypersurfaces represented by two equal time hyperplanes.
- It is convenient to expand the field in the basis of the boost modes

$$\psi_p(x,t) = \frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty} \mathrm{d}q \exp\left(\mathrm{i}m(x\sinh q - t\cosh q) - \mathrm{i}pq\right)$$

Klein-Gordon theory in Minkowski

All the relevant structures can be defined and the Hilbert space constructed.

- The complex structure results to be

$$J_{\Sigma_i} = \frac{\partial_t}{\sqrt{-\partial_t^2}}$$

- The vacuum state is the standard Minkowski vacuum state
- Amplitude and observable maps are implementable in terms of $\omega(\cdot,\cdot), g(\cdot,\cdot)$ and $\{\cdot,\cdot\}$

Klein-Gordon theory in Rindler space

- Rindler space is defined by $ds^2 = \rho^2 d\eta^2 - d\rho^2$, where

$$t = \rho \sinh \eta, \qquad x = \rho \cosh \eta$$

  It corresponds to the right wedge of Minkowski space, $\mathscr{R} := \{x \in \mathscr{M} : x^2 \leq 0, x > 0\}$.

- We consider the region $R \subset \mathscr{R}$ bounded by the disjoint union of two equal-Rindler-time hyperplanes.
- The field is expanded in the basis of the Fulling modes

$$\phi_p^R(\rho, \eta) = \frac{(\sinh(p\pi))^{1/2}}{\pi} K_{ip}(m\rho) e^{-ip\eta}, \qquad p > 0,$$

  K_{ip} is the modified Bessel function of the second kind (Macdonald function).

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- Amplitude and observable maps are implementable in terms of $\omega(\cdot,\cdot), g(\cdot,\cdot)$ and $\{\cdot,\cdot\}$

Boundary condition

In order for the quantum theory to be **well defined** the following condition must be imposed

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The relevance of this boundary condition manifests at the level of the algebraic structures, e.g.

$$\omega_{\Sigma_0}^{(\mathscr{R})}(\phi, \phi') = \omega_{\Sigma_0^R}(\phi^R, \phi^{R\prime}) + \lim_{\epsilon \to 0} \mathrm{i} \int_0^{\epsilon} \mathrm{d}p \, \frac{\cosh(p\pi)}{\sinh(p\pi)} \left[\phi(p) \overline{\phi(p)}' - \overline{\phi(p)} \phi'(p) \right],$$

where Σ_0 hyperplane $t = 0$, Σ_0^R is the semi-hyperplane $\eta = 0$, $\Sigma_0^R = \Sigma_0 \cap \mathscr{R}$.

$\Rightarrow$ the two quantum theories, in Minkowski and in Rindler spaces, are inequivalent

Outline

Unruh effect

Two notions:

1. **Global Unruh effect**: Comparison of particle states in Minkowski and Rindler spaces, e.g.
   - Crispino et al.:
     «The Unruh effect is defined in this review as the fact that the usual vacuum state for QFT in Minkowski spacetime restricted to the right Rindler wegde is a thermal state.»
   - Jacobson, ”Introductory lectures on black hole thermodynamics”:
     «The essence of the Unruh effect is the fact that the density matrix describing the Minkowski vacuum, traced over the states in the region $z < 0$, is precisely a Gibbs state for the boost Hamiltonian at a temperature $T = 1/2\pi$.»
2. **Local Unruh effect**: Comparison of expectation values of local observables, namely observable with compact support both in Minkowski and Rindler space.

Global Unruh effect

- Because of the inequivalence between the QFTs, no direct identification of Minkowski quantum states with Rindler quantum states is possible.
- **There is no global Unruh effect!**
- Same critique of the Russian school of Belinskii et al.

Local Unruh effect

- We consider the Weyl observable

$$F(\phi) = \exp\left(\mathrm{i}\int \mathrm{d}^2x\,\mu(x)\phi(x)\right),$$

$\mu(x)$ has compact support in the interior of the right wedge $\mathscr{R}$. F is a well defined observable in both Minkowski and Rindler spaces.

- We compute the expectation value of F
  1. on the Minkowski vacuum state

$$K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}},$$

  where K_{0,Σ_i} is the Minkowski vacuum state in $\mathscr{H}_{\Sigma_i}$, $(i = 1,2)$, and

  2. on the Rindler mixed state

$$D = \prod_i (1 - \exp(-2\pi k_i)) \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!(2k_i)^{n_i}} \psi_{n_i} \otimes \overline{\psi_{n_i}},$$

  ψ_{n_i} is the Rindler state with n_i particles defined in $\mathscr{H}_{\Sigma_i^R}$, $(i = 1,2)$.

Local Unruh effect

Using the observable map we compute the two expectation values:

- Expectation value in Minkowski space

$$\rho_M^F(K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}}) = \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\,\mathrm{d}^2x'\; \mu(x) G_F^{\mathcal{M}}(x,x')\mu(x')\right),$$

where $G_F^{\mathcal{M}}(x,x')$ is the Feynman propagator in Minkowski.

Local Unruh effect

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where $G_F^{\mathcal{M}}(x,x')$ is the Feynman propagator in Minkowski.

- Expectation value in Rindler space

$$\rho_R^F(D) = \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!(2k_i)^{n_i}} N^{-2}\int \mathrm{d}\xi_1\,\mathrm{d}\overline{\xi_1}\,\mathrm{d}\xi_2\,\mathrm{d}\overline{\xi_2}\,\rho_R^F(K_{\xi_1}\otimes\overline{K_{\xi_2}})$$
$$\exp\left(-\frac{1}{2}\int\frac{\mathrm{d}k}{2k}|\xi_1(k)|^2\right)(\xi_1(k_i))^{n_i}\exp\left(-\frac{1}{2}\int\frac{\mathrm{d}k}{2k}|\xi_2(k)|^2\right)(\overline{\xi_2(k_i)})^{n_i},$$

where the n-particle states have been expanded in the basis of the coherent states K_{ξ_i}

Local Unruh effect

The result of the computation is

$$\boxed{\rho_M^F(K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}}) = \rho_R^F(D)}$$

The local Unruh effect exists!

Local Unruh effect

The result of the computation is

$$\boxed{\rho_M^F(K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}}) = \rho_R^F(D)}$$

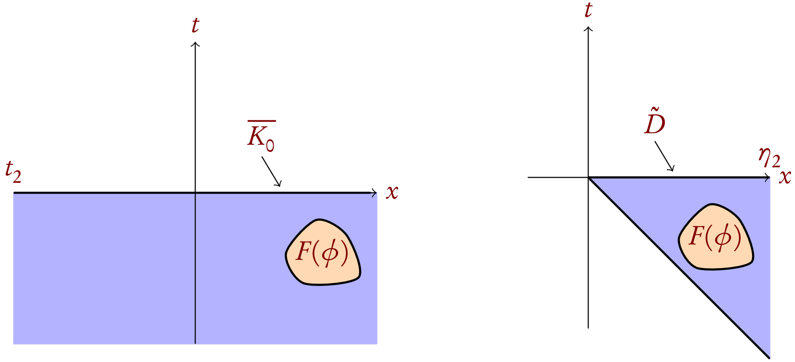
The local Unruh effect exists!

Consider a generic mixed state $\tilde{D}$ expanded in the basis of coherent states. Imposing the equality of expectation values for the theories in Minkowski and in Rindler fixes the coefficients of the state $\tilde{D}$, and the result is $\tilde{D} = D$.

$$\boxed{\text{the state } D \text{ is unique.}}$$

Regions with one boundary hypersurface

We consider other regions in Minkowski, $[t_1 = -\infty, t_2 = 0] \times \mathbb{R}$, and in Rindler, $[\eta_1 = -\infty, \eta_2 = 0] \times \mathbb{R}$



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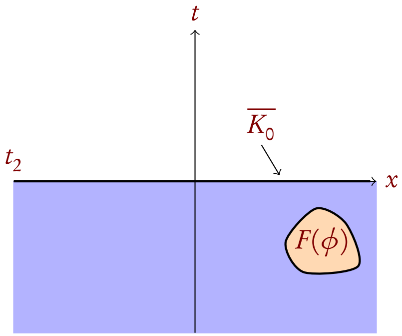

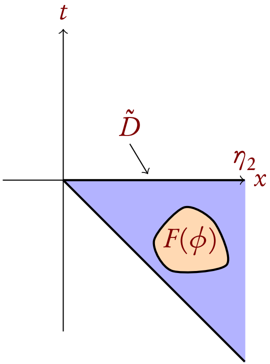

$$\boxed{\rho^F_M(\overline{K_0}) \neq \rho^F_R(\tilde{D})}$$

No local Unruh effect!

Outline

Conclusions and outlook

Conclusions

- Successfull implementation of the GBF in Rindler space
- New perspective on the Unruh effect: the distinction of the notions of global and local Unruh effect offers a clarification between different positions on the Unruh effect.
- First application of the amplitude map and implementation of the Berezin-Toeplitz quantization scheme (no Unruh effect within this quantization scheme).

Outlook

- Construction of the GBF for more general spacetime regions (in particular compact regions that avoide the origin of Minkowski spacetime)
- Composition of hypersurfaces and corresponding algebraic structures
- Relation with the Hawking effect