The positive formalism: spacetime and locality

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(Abstract) Positive Formalism

Obtain an **axiomatic framework** for encoding physical theories with:

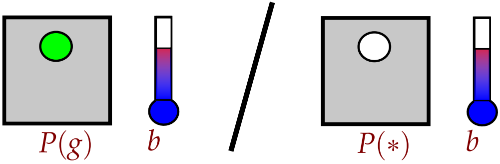
- **types**
  - a collection of **process types**
  - a collections of **interface types**
  - a **boundary map** from process types to interface types $M \mapsto \partial M$
- objects
  - a partially ordered vector space of **probes** $\mathcal{P}_M$ per process type M
  - a partially ordered vector space of **generalized boundary conditions** $\mathcal{B}_\Sigma$ per interface type Σ
- **compositions**
  - **decomposition** of interface types $\Sigma = \Sigma_1 \cup \cdots \cup \Sigma_n$ with associated positive isomorphism $\mathcal{B}_{\Sigma_1} \otimes \cdots \otimes \mathcal{B}_{\Sigma_n} \to \mathcal{B}_\Sigma$
  - **composition** of process types M and N to $M \cup N$ and probes $\diamond : \mathcal{P}_M \times \mathcal{P}_N \to \mathcal{P}_{M \cup N}$
- **values**: positive pairings $[\![\cdot, \cdot]\!]_M : \mathcal{P}_M \times \mathcal{B}_{\partial M} \to \mathbb{R}$

Measurement probabilities

For process type M consider the probe $P(*) \in \mathcal{P}_M^+$ encoding a measurement device and $P(g)$ encoding in addition a selected outcome.

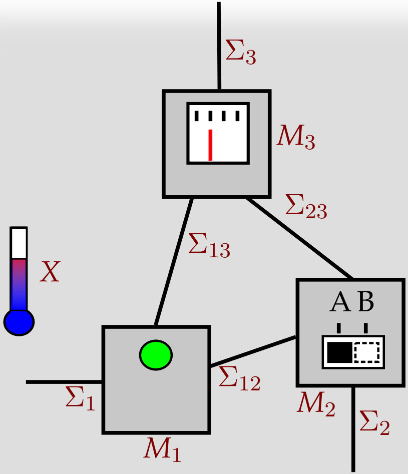
Given $b \in \mathcal{B}_{\partial M}^+$ the **probability** Π for an affirmative outcome is:

$$\Pi = \frac{[\![P(g), b]\!]_M}{[\![P(*), b]\!]_M}$$



Since $0 \leq P(g) \leq P(*)$ we have $0 \leq \Pi \leq 1$ (if $[\![P(*), b]\!]_M \neq 0$).

An example



M_1: light

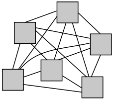
- $P(*)$ (apparatus)
- $P(r)$ (light red)
- $P(g)$ (light green)

M_2: switch

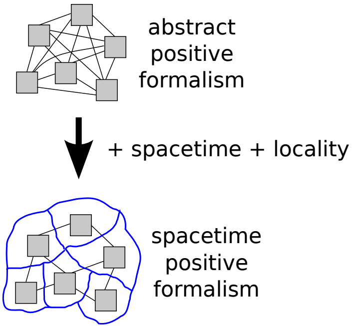
- $Q(A)$ (position A)
- $Q(B)$ (position B)

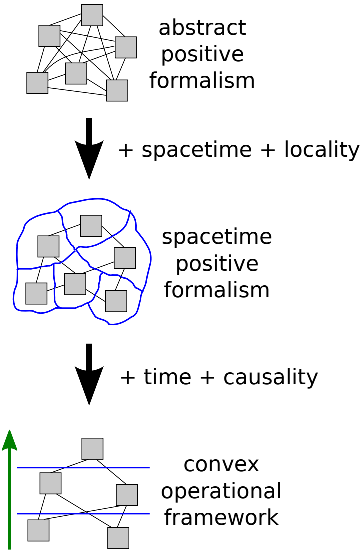
M_3: meter

- $R[*]$ (apparatus)
- $R[a, b]$ (range $[a, b]$)
- R (reading)

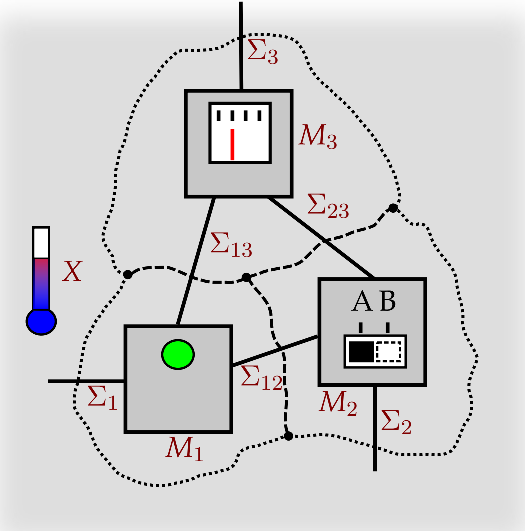


abstract
positive
formalism


abstract positive formalism
+ spacetime + locality
spacetime positive formalism

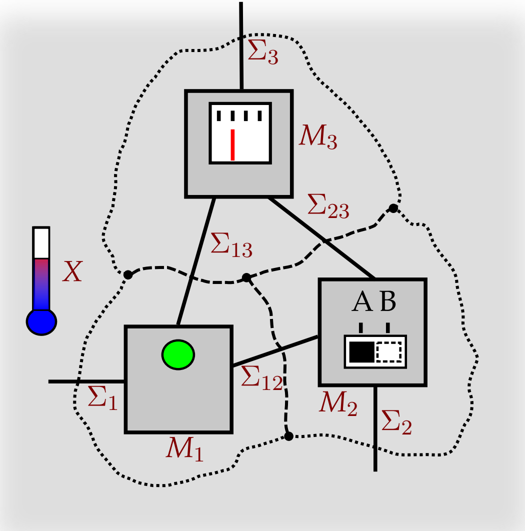

abstract positive formalism
+ spacetime + locality
spacetime positive formalism
+ time + causality
convex operational framework

Adding spacetime and locality



Spacetime locality provides a powerful organizing principle. Processes only interface with adjacent processes. This decreases considerably the inter-connectivity of the graph.

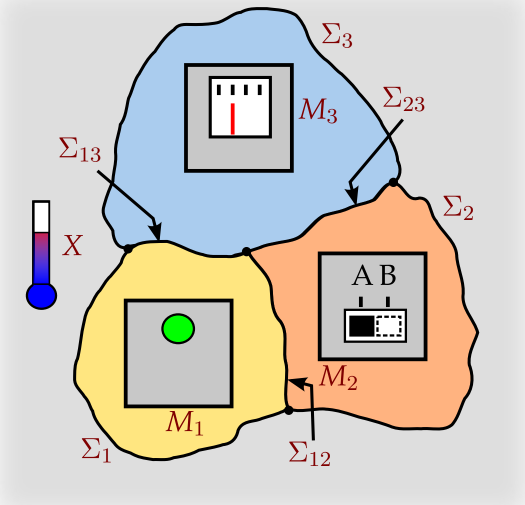
Adding spacetime and locality



Spacetime locality provides a powerful organizing principle. Processes only interface with adjacent processes. This decreases considerably the inter-connectivity of the graph.

We associate a spacetime **region** to any process and a **hypersurface** to any interface. These form a **dual complex** to the graph of boxes and links.

Adding spacetime and locality

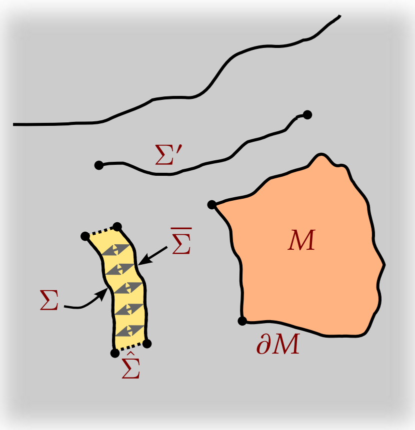


We may forget about the graph and identify **process types** with **regions** and **interface types** with **hypersurfaces**.

This framework is called the **local positive formalism**.

TQFT – manifolds

Fix dimension d. Manifolds are **oriented** and may carry additional structure: differentiable, metric, complex, etc.



region M

d-manifold with boundary.

hypersurface Σ

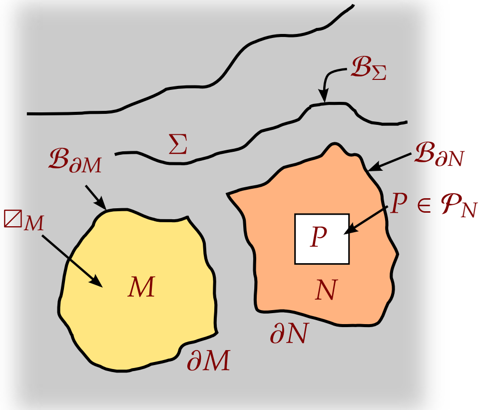
$d-1$-manifold with boundary, with germ of d-manifold.

slice region $\hat{\Sigma}$

$d-1$-manifold with boundary, with germ of d-manifold, interpreted as "infinitely thin" region.

positive TQFT – axioms I

Manifolds need not be oriented.



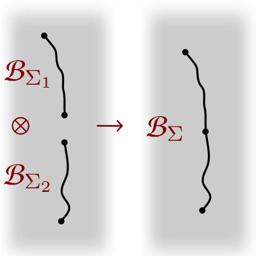
(P1) per hypersurface Σ

A partially ordered vector space $\mathcal{B}_\Sigma$.

(P4) per region M

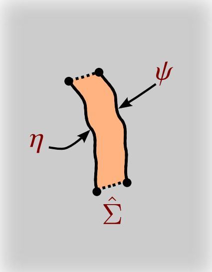
A partially ordered vector space $\mathcal{P}_M$ of linear maps $\mathcal{B}_{\partial M} \to \mathbb{R}$. $\mathcal{P}_M^+ \subset \mathcal{P}_M$ are **positive maps**. A **unit** $\boxslash_M \in \mathcal{P}_M^+$.

The choice of an element of $\mathcal{P}_M$ for a region M is indicated by a **label**.



(P2) per hypersurface decomposition $\Sigma = \Sigma_1 \cup \Sigma_2$

A positive vector space isomorphism $\tau : \mathcal{B}_{\Sigma_1} \otimes \mathcal{B}_{\Sigma_2} \rightarrow \mathcal{B}_{\Sigma}$.



(P3x) per hypersurface Σ

The unit gives rise to a **positive-definite sharply positive inner product** $(\psi, \eta)_\Sigma := \boxslash_{\hat{\Sigma}} \circ \tau(\psi \otimes \eta)$.

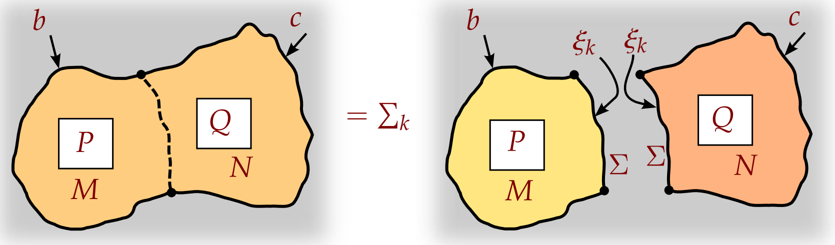
positive TQFT – axioms III

(P5a) per disjoint composition of regions $M = M_1 \sqcup M_2$

$\diamond : \mathcal{P}_{M_1} \times \mathcal{P}_{M_2} \to \mathcal{P}_M$ given by $(P_1 \diamond P_2)(\tau(\psi_1 \otimes \psi_2)) = P_1(\psi_1) P_2(\psi_2)$.
Also $\boxslash_{M_1} \diamond \boxslash_{M_2} = \boxslash_M$.

(P5b) per self-composition of region M to M_1 along Σ

$\diamond : \mathcal{P}_M \to \mathcal{P}_{M_1}$ given by $(\diamond P)(\psi) = \sum_k P(\tau(\psi \otimes \xi_k \otimes \xi_k))$. $\diamond \boxslash_M = \boxslash_{M_1}$.



Here, $\{\xi_k\}_{k \in I}$ is an orthonormal basis of $\mathcal{B}_\Sigma$.

Main reference

R. O., *A local and operational framework for the foundations of physics*, `arXiv:1610.09052`.