The positive formalism: causality and arrow of time

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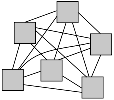
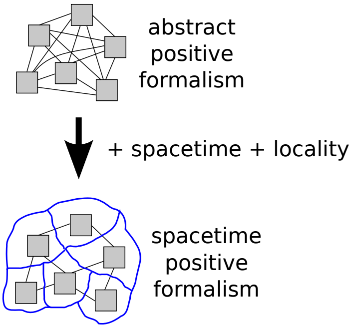
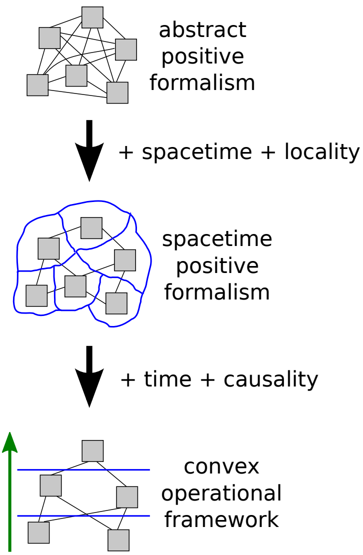


abstract
positive
formalism


abstract positive formalism
+ spacetime + locality
spacetime positive formalism


abstract positive formalism
+ spacetime + locality
spacetime positive formalism
+ time + causality
convex operational framework

Review: Time-evolution

Specialize to a global factorizing spacetime $\mathbb{R} \times \Sigma$ and restrict the spacetime system to **equal-time hyperplanes** Σ_t and **time-interval regions** $[t_1, t_2] = [t_1, t_2] \times \Sigma$.

Write $\mathcal{B}_t := \mathcal{B}_{\Sigma_t}$ and call this the (generalized) **state space** at time t.

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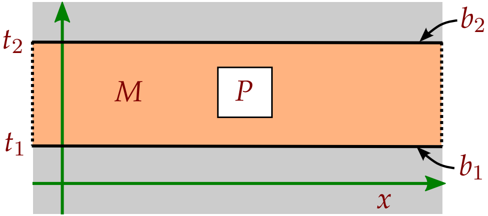

Consider probe $P \in \mathcal{P}_{[t_1,t_2]}$. Define the **probe map** $\tilde{P} : \mathcal{B}_{t_1} \to \mathcal{B}_{t_2}$ via

$$(\!| b_2, \tilde{P}(b_1) |\!)_{t_2} = [\![P, b_1 \otimes b_2]\!]_{[t_1,t_2]}, \qquad \forall b_1 \in \mathcal{B}_{t_1}, b_2 \in \mathcal{B}_{t_2}.$$

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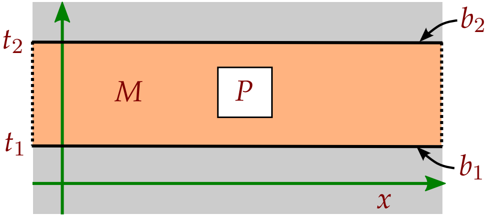

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That is, $\tilde{P}(b) = \sum_{k \in I} [\![P, b \otimes \xi_k]\!]_{[t_1,t_2]} \xi_k$.

The state of maximal uncertainty

Recall that the **boundary conditions** form a **hierarchy of generality**.

We assume that there exists a state $\mathbf{e} \in \mathcal{B}^+$ that is maximally general, call this the **state of maximal uncertainty**. This encodes a complete lack of knowledge.

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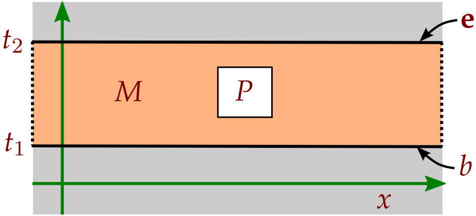
Most often in a measurement, we are only interested in the outcome given a fixed initial state b_1, but do not care about the state after the measurement.

This is encoded by setting the final state $b_2 = \mathbf{e}$.

Measurement without post-selection

Consider a binary measurement in $[t_1, t_2]$ encoded by a **non-selective probe** Q and a **selective probe** P.



The probability Π for an affirmative outcome given an initial state $b \in \mathcal{B}$, but disregarding the final fate of the system is thus,

$$\Pi = \frac{[\![P, b \otimes \mathbf{e}]\!]_{[t_1,t_2]}}{[\![Q, b \otimes \mathbf{e}]\!]_{[t_1,t_2]}} = \frac{(\!|\mathbf{e}, \tilde{P}(b)|\!)}{(\!|\mathbf{e}, \tilde{Q}(b)|\!)}.$$

One also says that this is a measurement **without post-selection**.

Normalization

The **positivity** and **positive-definiteness** of the inner product implies that for any $b \in \mathcal{B}^+$ with $b \neq 0$ we have $(\!|\mathbf{e}, b|\!) > 0$.

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This suggests corresponding notions for probe maps $\tilde{P} : \mathcal{B} \to \mathcal{B}$.

$\tilde{P} : \mathcal{B} \to \mathcal{B}$ is **normalization preserving** iff $(\!|\mathbf{e}, \tilde{P}(b)|\!) = (\!|\mathbf{e}, b|\!)$ for all $b \in \mathcal{B}$.

$\tilde{P} : \mathcal{B} \to \mathcal{B}$ is **normalization decreasing** iff $(\!|\mathbf{e}, \tilde{P}(b)|\!) \leq (\!|\mathbf{e}, b|\!)$ for all $b \in \mathcal{B}$.

Causality

Require that **non-selective probe maps** are **normalization preserving**.

This implies that **selective probe maps** are **normalization decreasing**.

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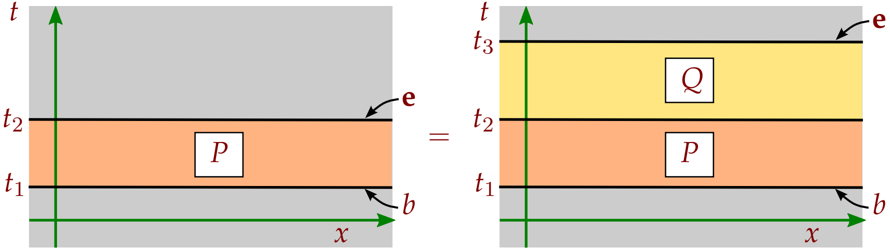
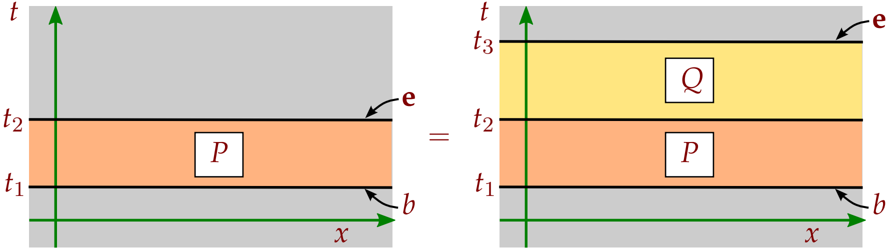
This turns out to implement **(forward) causality**.

Causality



Consider a binary measurement given by non-selective probe P_* and selective probe P_r. Possibly, a later measurement is performed given by non-selective probe Q.

Causality



Consider a binary measurement given by non-selective probe P_* and selective probe P_r. Possibly, a later measurement is performed given by non-selective probe Q.

The probability Π of an affirmative outcome of the first measurement does not depend on the second measurement being performed or not.

$$\Pi = \frac{\llparenthesis \mathbf{e}, \tilde{P}_r(b) \rrparenthesis}{\llparenthesis \mathbf{e}, \tilde{P}_*(b) \rrparenthesis} = \frac{\llparenthesis \mathbf{e}, \tilde{Q}(\tilde{P}_r(b)) \rrparenthesis}{\llparenthesis \mathbf{e}, \tilde{Q}(\tilde{P}_*(b)) \rrparenthesis}$$

Note: If b is normalized the denominators are equal to 1.

Time-asymmetry

The normalization conditions for probe maps are **time-asymmetric**. The normalization preserving condition,

$$(\!|\mathbf{e}, b|\!) = (\!|\mathbf{e}, \tilde{P}(b)|\!) \qquad \forall b \in \mathcal{B}$$

reads in the general formalism as,

$$[\![\boxslash, b \otimes \mathbf{e}]\!] = [\![P, b \otimes \mathbf{e}]\!] \qquad \forall b \in \mathcal{B}.$$

The time-reversed condition reads,

$$[\![\boxslash, \mathbf{e} \otimes b]\!] = [\![P, \mathbf{e} \otimes b]\!] \qquad \forall b \in \mathcal{B}.$$

Classical and quantum theory

If we take $\mathcal{B}$ to be a **lattice**, i.e., the space of real valued functions on a set (**phase space**) we obtain **classical statistical mechanics**.

- $\mathbf{e} = \mathbf{1}$ the constant function with value 1.
- $(\!|\cdot,\cdot|\!) = \int \cdot \cdot \, \mathrm{d}\mu$ the L^2 **inner product**
- certain **probe maps** describe **observables**, others describe modified dynamics

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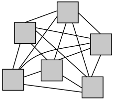
- certain **probe maps** describe **observables**, others describe modified dynamics

If we take $\mathcal{B}$ to be an **anti-lattice**, i.e., the space of self-adjoint operators on a Hilbert space $\mathcal{H}$, we obtain **quantum statistical mechanics**.

- $\mathbf{e} = \mathrm{id}_{\mathcal{H}}$ the **identity operator**
- $(\!|\cdot,\cdot|\!) = \mathrm{tr}(\cdot\cdot)$ the **Hilbert-Schmidt inner product**
- **primitive probe maps** are **quantum operations**

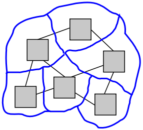
classical
(lattices)

quantum
(anti-lattices)



abstract positive formalism

+ spacetime + locality



spacetime positive formalism

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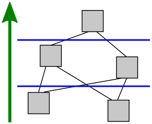
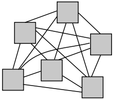
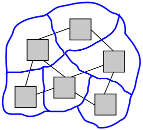
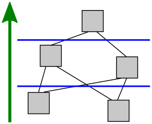
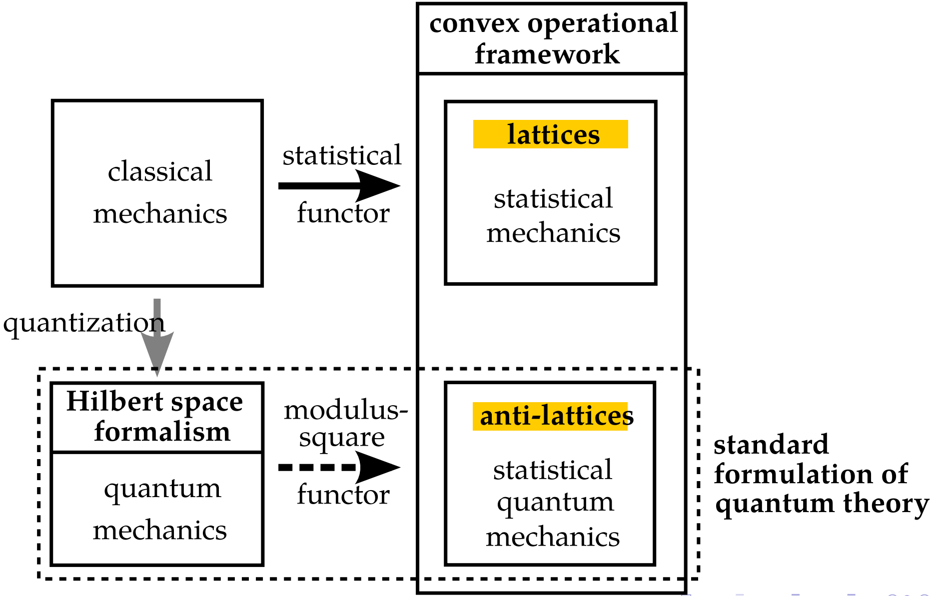


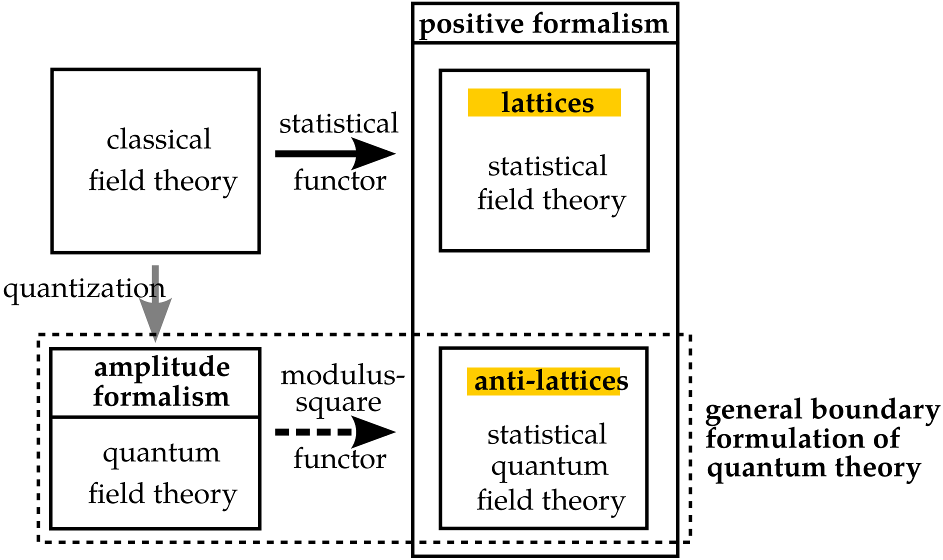
convex operational framework

classical (lattices)		**quantum** (anti-lattices)
abstract classical statistical theory	 abstract positive formalism	abstract quantum theory
	+ spacetime + locality	
spacetime statistical field theory	 spacetime positive formalism	general boundary formulation / axiomatic QFT
	+ time + causality	
statistical mechanics	 convex operational framework	standard formulation of quantum theory

Time-evolution frameworks



Spacetime frameworks



Main reference

R. O., *A local and operational framework for the foundations of physics*, `arXiv:1610.09052`.