Operational quantum gravity: black hole to white hole transition

Robert Oeckl

Centro de Ciencias Matemáticas
Universidad Nacional Autónoma de México
Morelia, Mexico

Seminar *General Boundary Formulation*
31 October 2018

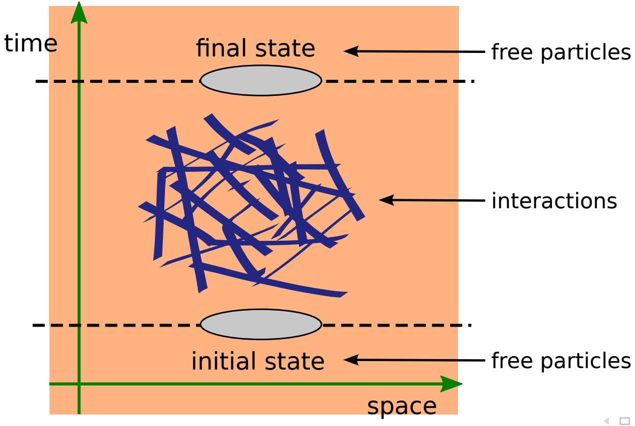
What is a measurement in quantum gravity?

The standard formulation of quantum theory is based on a non-relativistic notion of spacetime. Time plays a special role. It fails to make sense in a general relativistic setting.

To apply the standard formulation we need to restrict the measurement to occur in classical regions of spacetime, where the metric is frozen.

Asymptotic measurement: QFT

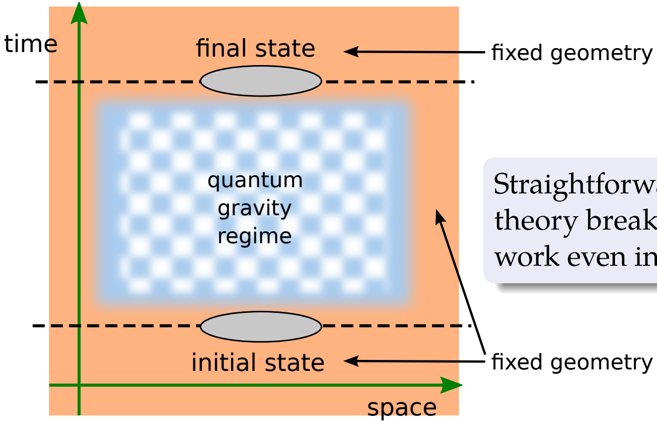
Consider measurement only at **asymptotic infinity**, infinitely early and infinitely late time, described by **transition probabilities**. This is how the **S-matrix** in **quantum field theory** works to describe **scattering processes**. This requires **perturbation theory**.



At early and late times particles are far apart and do not interact. The interesting physics happens at **intermediate times**.

Asymptotic measurement: QG ?

Fix an approximate **classical metric background** at **asymptotic infinity**. Observations take place exclusively in this region. This requires **perturbation theory** in the **metric**.
(Perturbative Quantum Gravity, String Theory)



Straightforward perturbation theory breaks down. Can this work even in principle?

A bouncing black hole

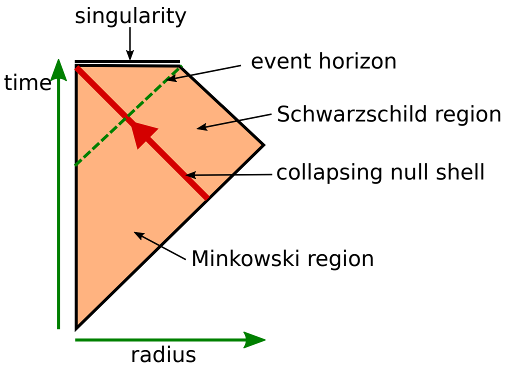
To illustrate the limitations of the asymptotic approach we consider a model for a bouncing black hole.

A bouncing black hole

To illustrate the limitations of the asymptotic approach we consider a model for a bouncing black hole.

We then show how the positive formalism allows us to make predictions in this case.

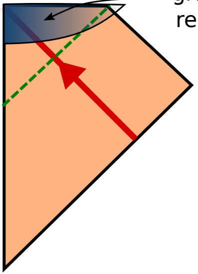
Black hole



Consider a black hole that is formed from a **thin shell** that **contracts** at the **speed of light**. We can imagine this to consist of photons.

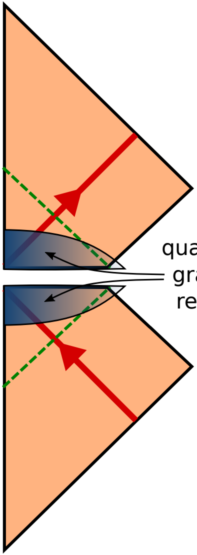
There is one parameter describing this setting: The **mass** m of the shell.

Black hole and white hole



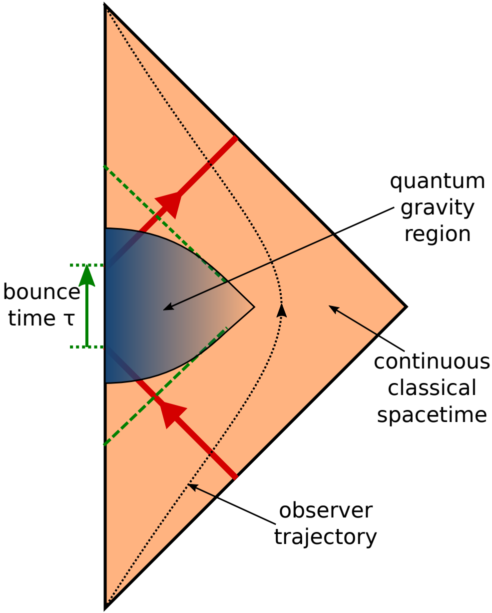
We suppose that quantum gravity becomes relevant at high curvature and prevents the formation of a singularity.

Black hole and white hole



We suppose that quantum gravity becomes relevant at high curvature and prevents the formation of a singularity. Instead a **bounce** occurs and a **white hole** is formed. The mass is ejected into a thin shell expanding at the speed of light.

Bouncing black hole



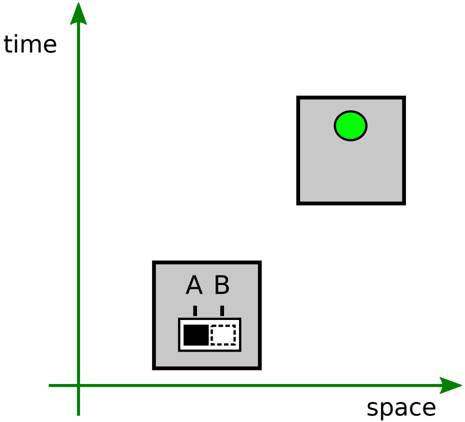
The black hole and white hole spacetimes can be sewn together so that there is a continuous classical spacetime outside the quantum gravity region.

There is one free parameter: The **bounce time** τ.

How to predict the bounce time?

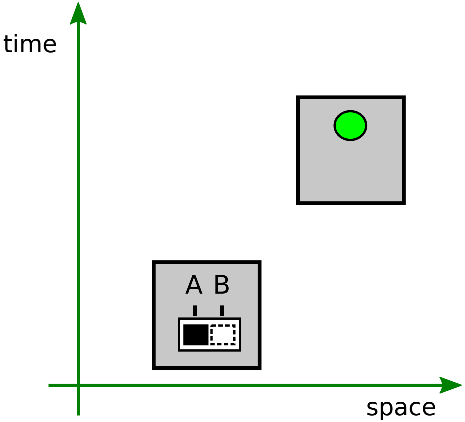
This question is outside of the scope of the asymptotic measurement setup. There is no single fixed asymptotic metric. Rather, the asymptotic metric is different for each bounce time. On the other hand, the incoming and outgoing particles always seem to be the same.

Elements of the positive formalism



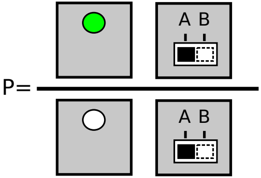
Consider measurements, observations, interventions etc. distributed in spacetime.

Elements of the positive formalism

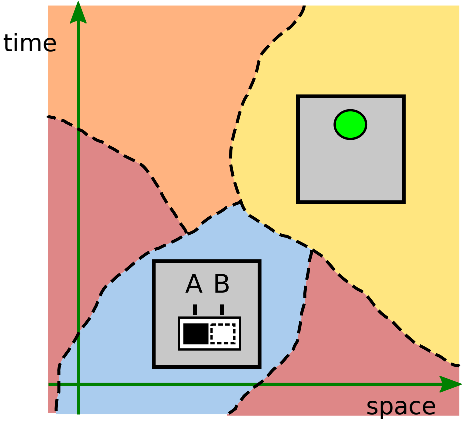


Consider measurements, observations, interventions etc. distributed in spacetime.

We wish to predict **correlations** between them. For example: The **probability** P for the light to turn green given that we put the switch in position "A":



Elements of the positive formalism

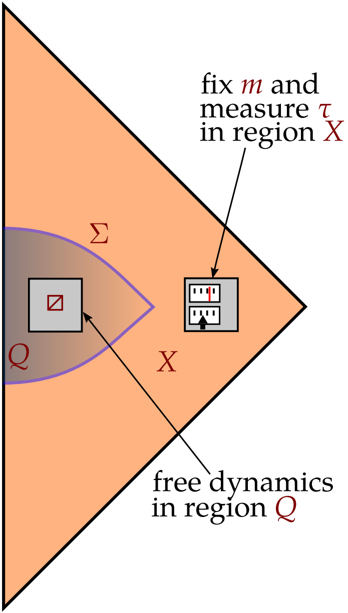


For a **local** description cut up spacetime into pieces, called **regions**. These are in contact with each other through **hypersurfaces**.

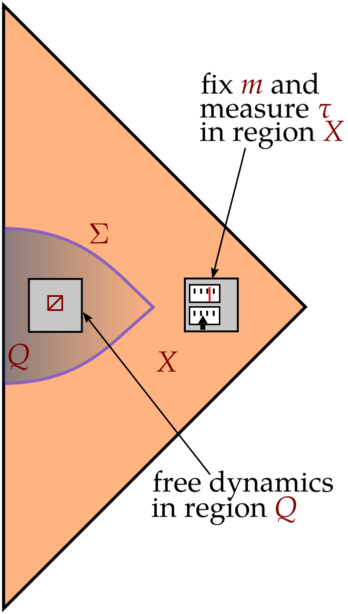
Associate **processes** (*probes*) to regions to encode experiments etc. **Compose** processes along with the underlying regions.

Associate **states** (*boundary conditions*) to hypersurfaces to parametrize **interactions**.

Bounce model: bounce time probability

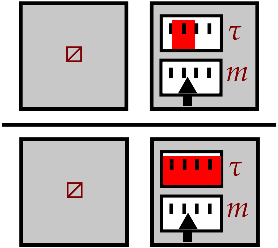


Bounce model: bounce time probability



Given a fixed shell mass m_0, we wish to determine the **probability** for the bounce time τ to lie in a given interval $[\tau_1, \tau_2]$. This is,

$P(m_0, [\tau_1, \tau_2]) =$



In formal notation,

$$P(m_0, [\tau_1, \tau_2]) = \frac{[\![\boxslash_Q, b_X[m_0, [\tau_1, \tau_2]]]\!]}{[\![\boxslash_Q, b_X[m_0, [0, \infty]]]\!]}$$

Bounce model: classical

Suppose we had a **classical** theory of gravity allowing for the bounce.
For each **shell mass** m it assigns a **bounce time** $\tau_c(m)$.

Bounce model: classical

Suppose we had a **classical** theory of gravity allowing for the bounce. For each **shell mass** m it assigns a **bounce time** $\tau_c(m)$.

The **phase space** $L_\Sigma = \mathbb{R}^+ \times \mathbb{R}^+ = \{(m, \tau)\}$ at Σ is parametrized by mass m and bounce time τ. The space $\mathcal{B}_\Sigma$ of **boundary conditions** or **states** consists of **statistical distributions** on L_Σ, i.e., positive functions.

The **boundary conditions** $b_X[*]$ take the form,

$$b_X[m_0, [0, \infty]](m, \tau) = \delta(m - m_0),$$
$$b_X[m_0, [\tau_1, \tau_2]](m, \tau) = \delta(m - m_0)\chi_{[\tau_1,\tau_2]}(\tau).$$

Bounce model: classical

Suppose we had a **classical** theory of gravity allowing for the bounce. For each **shell mass** m it assigns a **bounce time** $\tau_c(m)$.

The **phase space** $L_\Sigma = \mathbb{R}^+ \times \mathbb{R}^+ = \{(m, \tau)\}$ at Σ is parametrized by mass m and bounce time τ. The space $\mathcal{B}_\Sigma$ of **boundary conditions** or **states** consists of **statistical distributions** on L_Σ, i.e., positive functions.

The **boundary conditions** $b_X[*]$ take the form,

$$b_X[m_0, [0, \infty]](m, \tau) = \delta(m - m_0),$$
$$b_X[m_0, [\tau_1, \tau_2]](m, \tau) = \delta(m - m_0)\chi_{[\tau_1, \tau_2]}(\tau).$$

The probe $\boxslash_Q$ enforces the classical bounce time,

$$[\![\boxslash_Q, f]\!] = \int f(m, \tau)\delta(\tau - \tau_c(m))\, \mathrm{d}m\, \mathrm{d}\tau = \int f(m, \tau_c(m))\, \mathrm{d}m.$$

As expected, this yields $P(m_0, [\tau_1, \tau_2]) = \chi_{[\tau_1, \tau_2]}(\tau_c(m_0))$.

Bounce model: quantum

Instead of the phase space L_Σ we have a **Hilbert space** $\mathcal{H}_\Sigma$. The space $\mathcal{B}_\Sigma$ of **boundary conditions** or **states** is now the space of **density matrices** on $\mathcal{H}_\Sigma$, i.e., positive operators.

Bounce model: quantum

Instead of the phase space L_Σ we have a **Hilbert space** $\mathcal{H}_\Sigma$. The space $\mathcal{B}_\Sigma$ of **boundary conditions** or **states** is now the space of **density matrices** on $\mathcal{H}_\Sigma$, i.e., positive operators.

We assume that we have **coherent states** $K_{m,\tau} \in \mathcal{H}_\Sigma$ that approximately describe the classical geometries in X corresponding to a given **shell mass** m and **bounce time** τ. These satisfy a **completeness relation**,

$$\mathbf{1} = \int |K_{m,\tau}\rangle\langle K_{m,\tau}|\,\alpha(m,\tau)\,\mathrm{d}m\,\mathrm{d}\tau.$$

Bounce model: quantum

Instead of the phase space L_Σ we have a **Hilbert space** $\mathcal{H}_\Sigma$. The space $\mathcal{B}_\Sigma$ of **boundary conditions** or **states** is now the space of **density matrices** on $\mathcal{H}_\Sigma$, i.e., positive operators.

We assume that we have **coherent states** $K_{m,\tau} \in \mathcal{H}_\Sigma$ that approximately describe the classical geometries in X corresponding to a given **shell mass** m and **bounce time** τ. These satisfy a **completeness relation**,

$$\mathbf{1} = \int |K_{m,\tau}\rangle\langle K_{m,\tau}| \, \alpha(m,\tau) \, \mathrm{d}m \, \mathrm{d}\tau.$$

This gives rise to a **positive operator valued measure (POVM)**. This allows us to **quantize** a classical **statistical distribution** f on phase space into a corresponding **mixed state** $\hat{f}$ of the quantum theory,

$$\hat{f} = \int f(m,\tau) |K_{m,\tau}\rangle\langle K_{m,\tau}| \, \alpha(m,\tau) \, \mathrm{d}m \, \mathrm{d}\tau$$

In this way we obtain, $\hat{b}_X[m_0, [0,\infty]]$ and $\hat{b}_X[m_0, [\tau_1, \tau_2]]$ as positive operators on $\mathcal{H}_\Sigma$ representing mixed states.

Bounce model: quantum

The **null probe** $\hat{\boxtimes}_Q$ in the quantum theory is determined by the **amplitude map** $\rho_Q : \mathcal{H}_\Sigma \to \mathbb{C}$ from the quantum gravity model.

Using the coherent states,

$$[\![\hat{\boxtimes}_Q, \sigma]\!] = \int \rho_Q(\sigma K_{m,\tau}) \, \overline{\rho_Q(K_{m,\tau})} \, \alpha(m, \tau) \, \mathrm{d}m \, \mathrm{d}\tau$$

Bounce model: quantum

The **null probe** $\hat{\boxslash}_Q$ in the quantum theory is determined by the **amplitude map** $\rho_Q : \mathcal{H}_\Sigma \to \mathbb{C}$ from the quantum gravity model.

Using the coherent states,

$$[\![\hat{\boxslash}_Q, \sigma]\!] = \int \rho_Q(\sigma K_{m,\tau})\, \overline{\rho_Q(K_{m,\tau})}\, \alpha(m,\tau)\, \mathrm{d}m\, \mathrm{d}\tau$$

If $\sigma = \hat{f}$ arises as the **quantization** of a classical statistical distribution f,

$$[\![\hat{\boxslash}_Q, \hat{f}]\!] = \int f(m,\tau) |\rho_Q(K_{m,\tau})|^2\, \alpha(m,\tau)\, \mathrm{d}m\, \mathrm{d}\tau.$$

Bounce model: quantum

The **null probe** $\hat{\boxslash}_Q$ in the quantum theory is determined by the **amplitude map** $\rho_Q : \mathcal{H}_\Sigma \to \mathbb{C}$ from the quantum gravity model.

Using the coherent states,

$$[\![\hat{\boxslash}_Q, \sigma]\!] = \int \rho_Q(\sigma K_{m,\tau}) \overline{\rho_Q(K_{m,\tau})} \, \alpha(m,\tau) \, \mathrm{d}m \, \mathrm{d}\tau$$

If $\sigma = \hat{f}$ arises as the **quantization** of a classical statistical distribution f,

$$[\![\hat{\boxslash}_Q, \hat{f}]\!] = \int f(m,\tau) |\rho_Q(K_{m,\tau})|^2 \, \alpha(m,\tau) \, \mathrm{d}m \, \mathrm{d}\tau.$$

With this we get,

$$P(m_0, [\tau_1, \tau_2]) = \frac{[\![\hat{\boxslash}_Q, \hat{b}_X[m_0, [\tau_1, \tau_2]]]\!]}{[\![\hat{\boxslash}_Q, \hat{b}_X[m_0, [0, \infty]]]\!]} = \frac{\int_{\tau_1}^{\tau_2} |\rho_Q(K_{m_0,\tau})|^2 \, \alpha(m_0,\tau) \, \mathrm{d}\tau}{\int_0^{\infty} |\rho_Q(K_{m_0,\tau})|^2 \, \alpha(m_0,\tau) \, \mathrm{d}\tau}.$$

References

R. O., *A predictive framework for quantum gravity and black hole to white hole transition*, Phys. Lett. A **82** (2018) 2622–2625, arXiv:1804.02428.